309

INDIAN JOURNAL OF THEORETICAL PHYSICS

Founder Director : Late Prof. K. C. Kar, D. Sc.

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SILVER JUBILEE VOLUME

(1953-1977)

INSTITUTE OF THEORETICAL PHYSICS
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SILVER JUBILEE VOLUME

SECTIONS I & II

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INDIAN JOURNAL OF THEORETICAL PHYSICS

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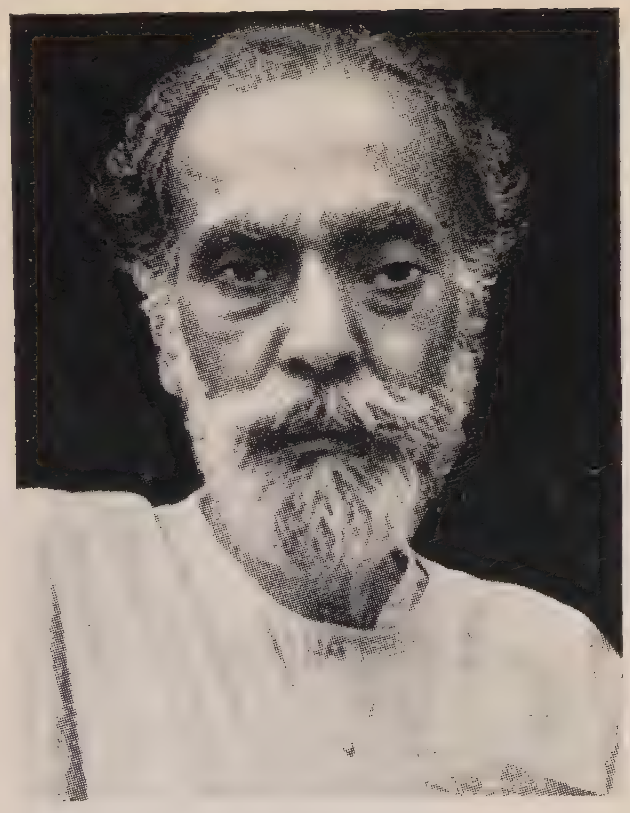
SECTION I



Prof. K. C. KAR, D. Sc.
Founder Director,
Institute of Theoretical Physics

Born :—
1st October 1899

Died :—
22nd April 1975

TWENTY FIVE YEARS

OF

INDIAN JOURNAL OF THEORETICAL PHYSICS

The year 1953 witnessed the foundation of the Indian Journal of Theoretical Physics. It was the brain-child of Prof. K. C. Kar, D. Sc., a theoretical Physicist of broad vision, whose curiosity in investigating new physical phenomena was matched only by the meticulous care with which he supervised the publication of his papers. He sponsored the publication of the Indian Journal of Theoretical Physics, the first of its kind in India, twenty five years ago, out of his personal resources.

Seen through the mist of years, the early struggles for the publication of this journal seems poignant to a degree. During his tenure as a Professor of Physics at the Presidency College, Calcutta. Dr. Kar had gathered around him a band of devoted workers, who worked for the fun of doing research work without any kind of remuneration. Among them, the most conspicuous were M. Ghosh, K. K. Mukherjee, A. Ganguli, D. Basu, R. R. Roy, S. Sanatani, S. Sen Gupta and others. Many of them came up above the horizon in their later lives. Their combined effort prepared the ground for a new sort of growth in the field of Theoretical Physics in India. Their early papers used to be published mostly in foreign journals like Philosophical Magazine, Zeitschift für Physik, Physikalische Zeitschrift für angewandt Physik, Physical Review etc. But such a procedure

turned out to be a time-consuming affair. Gradually the number of papers was transformed from a mere trickle into a spate and the local journals like the Indian Journal of Physics and the Bulletin of the Calcutta Mathematical Society could not cope with the situation, to ensure quicker publication. This led Professor Kar to think seriously about starting a new journal of his own, devoted entirely to Theoretical Physics. At this juncture Prof. J. Ghosh of the Department of Mathematics, Presidency College, Calcutta came to his rescue. He started the "Indian Physico-Mathematical Journal" which relieved the tension a great deal. Unfortunately, such a state of affair did not last long and the new journal soon ceased to function due to financial stringency. And so, the onus of starting another journal devolved upon the shoulders of Professor Kar. To begin with, he formed a small committee consisting of M. Ghosh, S. P. Banerjee and himself to go into details for such a venture. In order to reduce the printing cost, an erstwhile student to Dr. Kar was persuaded to print the first issue of the journal in his ramshackle Raghunath Press, Calcutta. But serious difficulty arose on account of dearth of necessary types of mathematical symbols. In fact, the rare ones had to be fabricated from discarded Bengali and Hindi letter types by filing away the redundant appendages.

The first issue of the Indian Journal of Theoretical Physics was published by the Editorial Secretary. The earliest board of Editors and members of the Advisory committee consisted of the following panel :

Dr. S. C. Kar, Ph. D.	Dr. K. C. Kar, D. Sc.
Dr. B. B. Sen, D. Sc.	Dr. M. Ghosh, D. Sc. (Hon. Secretary)

Subsequent issues were published by the Institute of Theoretical Physics, Calcutta.

Emerging from its preliminary teething troubles, the Indian Journal of Theoretical Physics gradually grew up in stature. Its sphere of circulation widened. It is now widely subscribed by universities and research institutions scattered all over the world, while the papers are regularly abstracted in Science Abstracts & Nuclear Science Abstracts. During the early decades, the printing of the journal was entrusted to Mudrani Press. This arrangement produced satisfactory results for a while till the printing costs became prohibitively high as the effect of inflation. At this turn of events, Dr. Kar became vaguely discontented. The visionary in him began to dream about purchasing a printing press for his own journal. One of his first tasks was to supervise the purchase and installation of a letter setting type equipment at the Institute. Two novices were trained up to compose the letter-setting manipulation at the Institute office and then get the impression from a neighbouring press machine. Meanwhile he had also placed an order for buying an indigenous treadle (machine). Unfortunately, this deal did not materialise because he passed away on April 22, 1975.

Inspite of hard times the Indian Journal of Theoretical Physics has maintained its effective performance for quarter of a century. It is now high time that we, his successors, should embark on an expanded programme and try to fulfil his legacy.

SECRETARY

INDIAN JOURNAL
OF
THEORETICAL PHYSICS

VOLUME 1

1953

The Classical interpretation of Dirac's theory of electron

K. C. Kar and S. Sanatani
(Presidency College, Calcutta, India)

ABSTRACT : An attempt has been made to explain Dirac's theory of electron without the help of matrices. For the linearisation of the relativistic Hamiltonian, a new method has been given. From physical consideration four wave functions are assigned to describe the motion of an electron in a central field. Then, on taking wave statistical averages, the Hamiltonian at once becomes linear and the four linear wave equations are obtained without any further assumptions. Matrices which "bring in" spin in Dirac theory are nowhere used in the present method and the spinning property of the electron has been taken into account through the spin-orbit interaction energy in the Hamiltonian.

The motion of a free electron is considered as a special case of the central field problem and 32 alternative sets of linear wave equations, all equally correct, are deduced in place of only one given by Dirac.

Dynamics of the vibration of a bar exhibiting strain-rate effect under conditions of longitudinal impact

Part III

S. K. Ghosh
(Chandernagore College, W. B., India)

ABSTRACT : Dynamics of the vibration of a bar exhibiting strain-rate effect and excited by an axial elastic impact has been fully developed following the powerful Operational method. Unlike other theories, the present theory is free from the assumption of any law of impact loading. The present theory explains the fluctuating nature of the stress-time variation at the impact end and marks the boundary of continuous transition from elastic to plastic behaviour. The results indicate satisfactory agreement between theory and experiment.

VOL. 1, NO. 1, 1953

On the elastic distortion of a cylindrical hole by localized axial shears on the inner boundary

Sisir Chandra Das
(Chandernagore College, W. B., India)

ABSTRACT : The effect of a localized axial shear as produced by a piston head on the inner surface of an infinitely long cylindrical hole in an elastic solid has been considered in this paper.

VOL. 1, NO. 1, 1953

On the distribution of stable nuclei

S. Sengupta
(Darjeeling Govt. College, W. B., India)

ABSTRACT : A few assumptions are made regarding the nature and magnitude of the variation of binding energy of the last proton and neutron in a nucleus. It is shown that these

assumptions are consistent with the Saturation character of the nuclear forces. Then these assumptions are applied to study the various systematics in the distribution of stable nuclei. It is found that all the regularities are satisfactorily explained with these assumptions.

VOL. 1, NO. 2, 1953

Classical derivation of the pseudoscalar interaction potential

K. C. Kar and H. Mukherjee
(Presidency College, Calcutta, India)

ABSTRACT: Following different methods, Bethe and Pauli have derived formulæ for the interaction potential between two nucleons. Although the pseudoscalar part is same in both, the scalar part is different in sign and magnitude. It is shown that both the formulæ of Bethe and Pauli may be derived from purely classical ideas of doublet-doublet interaction. The assumption of inseperability of poles leads to the other. As, however, the idea of seperability cannot be justified Bethe's formula is preferable to that of Pauli.

VOL. 1. NO. 2, 1953

Energy absorbed by a bar under a compressive impact by an elastic load

Part IV

Sudhir Kumar Ghosh
(Chandernagore College, W. B., India)

ABSTRACT: The theoretical expression for the energy absorbed by a bar subjected to impact at the free end, the other end being fixed, by an elastic load of any mass, is calculated with the help of the general theory developed by the author previously, in a series of papers.

The experimental results obtained by E. T. Habib who in course of his study on the high speed compression tests on small

Copper cylinders measured the energy absorbed by the cylinder,—have been fully interpreted from the theory. The effects of elasticity of the load and the 'mass-ratio' of the load and the bar system on the energy imparted to the bar, have been also studied.

VOL. 1, NO. 2, 1953

On the effect of shearing stresses applied on the boundary of a circular hole in a semi-infinite plate

Sisir Chandra Das

(Chandernagore College, W. B., India)

ABSTRACT : In this paper bipolar co-ordinates have been used to determine stresses due to applied tangential forces on the inner boundary of a circular hole near the straight edge of a semi-infinite plate.

VOL. 1 NO. 2, 1953

On a new theory of alpha-disintegration

K. C. Kar and S. K. Kundu

(Presidency College, Calcutta, India)

ABSTRACT : In the present paper the theory of alpha-decay previously given by Kar and Choudhury (1950) has been considerably improved and extended and far better agreement obtained with the experimental variation of log λ with energy of emission (E). From the proposed theory the calculated upper limit of E_{max}/E_{min} for nuclei emitting groups of alpha particle is 1·50, whereas the experimental value so far obtained is 1·44. This striking confirmation of the theory supports the assumption made regarding the nature of the extra-nuclear, short-range force, namely, that the meson field acting on the outgoing alpha particle continuously transforms into fields corresponding to lighter and lighter meson masses till it vanishes at the emission

radius (r_1). Accordingly, in the region between the nuclear radius r_0 and r_1 the kinetic energy of the outgoing alpha-particle is positive unlike Gamow's theory which predicts negative kinetic energy and so imaginary velocity in this region.

Owing to this fundamental difference the present theory gives a formula for the disintegration constant involving an important cosine factor in place of the exponential Gamow factor in Gamow's formula. Because the angle is nearly $\pi/2$ the logarithm of the cosine factor has wide variations with energy and the agreement with experiment is found to be decidedly better. The fact that cosine factor, occuring in the wave function for the alpha particle, is nearly zero at the nuclear surface, shows that the nuclear surface is nearly a density mode and so a velocity antinode.

VOL. 1, NO. 3, 1953

Note on love waves in a homogeneous crust laid upon heterogeneous medium (II)

Sushil Chandra Dasgupta
(Krishnagar College, W. B., India)

ABSTRACT : It has been shown in this paper that propagation of Love waves is possible in a homogeneous crust lying on a heterogeneous medium in which both rigidity and density vary as $\cosh^2\left(\frac{z}{\lambda}\right)$, λ being a constant and z, the depth below the surface of the substratum.

VOL. 1, NO. 3 1953

Radial and Torsional vibrations of a cylindrically aeolotropic annulus

A. M. Sengupta
(B. E. College, Howrah, W. B., India)

ABSTRACT : In this paper free radial and torsional vibrations of a circular annulus having small thickness and possessing the same type of cylindrical aeolotropy as considered by Carrier (1943) have been discussed.

VOL. 1 NO. 3, 1953

On beta emission and energy levels of the product nuclei

Satyaprabhat Banerjee and (Miss) Amita Mitra
(Presidency College, Calcutta, India)

ABSTRACT : An attempt has been made to give systematic level schemes for the beta active nuclei in the region $Z=81$ to $Z=92$ for the naturally occurring heavy radioactive isotopes. The total beta decay energy thus computed has been verified by considering (1) alpha beta branching and closed cycles in a series (2) the relation between alpha energy and the binding energy of last proton (3) magic number drop of the binding energy of last proton and (4) the decay energy graph. A few predictions have been made regarding some undetermined beta energies.

VOL. 1, NO. 4, 1953

On the stresses due to a small spherical inclusion in an elastic solid under uniform shearing stress

Sisir Chandra Das
(Chandernagore College, W. B., India)

ABSTRACT : In this paper, the concentration of stresses in an isotropic elastic solid due to a small spherical inclusion under uniform shearing stress has been found. The results are discussed for several particular cases.

VOL. 1, NO. 4, 1953

VOLUME 2

1954

On the decay energy in artificial disintegration

Mrs. Rekha Basu
(Presidency College, Calcutta, India)

ABSTRACT : Kar and Kundu's theory of spontaneous alpha disintegration is extended and applied to the process of artificial disintegration by considering the recoil of the residual nucleus.

Modified formulæ have been obtained for the upper limits of $(E_2)_{max}/(E_2)_{min}$ for the two cases (*i*) $\xi_1 > 0$ and (*ii*) $\xi_1 < 0$, where ξ_1 is the total kinetic energy in *C*-system at the emission radius r_1. The formulæ obtained have been shown to give values in close agreement with the experimental values of the upper limits of $(E_2)_{max}/(E_2)_{min}$ for different reactions.

VOL. 2, NO. 1, 1954

The generalised interaction potential between nucleons

K. C. Kar and H. Mukherjee
(Presidency College, Calcutta, India)

ABSTRACT : Introducing the concept of nucleonic magnetic moment and corresponding magnetic field, a generalised formula has been derived for the short range interaction potential between two nucleons by taking into account the spin-spin and spin-orbit effects. The effect of the spin of the emitted meson on the interaction potential is also discussed. It is shown that the idea of a short range static potential leads to the pseudovector potential derived by Kemmer.

VOL. 2, NO. 1, 1954

A note on Sargent's rule in β-decay

Miss Amita Mitra
(Presidency College, Calcutta, India)

ABSTRACT : Elements in the regions $30 \leqslant Z \leqslant 35$, $50 \leqslant Z \leqslant 55$ and $82 \leqslant Z \leqslant 92$ have been studied. It is shown that instead of drawing curves for $\log \lambda - \log E_{max}$ as by Sargent, curves should be drawn for $\log \lambda - E_{max}$ for given Z values. In that case the curves arc shown to be perfectly smooth having a general upward trend. Elements giving complex spectra are also found to follow this general upward trend when the lowest value of E_{max} is considered. However, for magic numbers $Z=50$ and $Z=82$ double trends have been found, being upward for even neutron number but downward for odd neutron number.

VOL. 2, NO. 1, 1954

Some problems of elastic plates containing circular holes II

A. M. Sengupta
(B. E. College, Howrah, W. B., India)

ABSTRACT : In this paper the influence of stresses around a circular hole in a semi-infinite plate subjected to a couple M applied on its straight edge is discussed.

VOL. 2, NO. 1, 1954

Note on the minimum bowing pressure

K. C. Kar
(Presidency College, Calcutta, India)

VOL. 2, NO. 1, 1954

A simple derivation of Klein-Nishina formula without matrices

K. C. Kar, S. Sanatani and R. K. Bhattacharyya
(Presidency College, Calcutta, India)

ABSTRACT : Klein-Nishina formula for differential cross-section of Compton scattering, so far deduced only on the basis of Dirac's theory of electron is derived here by an alternative method which does not involve Dirac's non-commutative matrices. The lengthy and intricate 'spur' calculation has been avoided and it is shown how the exact formula can be obtained by the ordinary process of summation.

On using the linear form of the relativistic Hamiltonian recently obtained by Kar and Sanatani (Ind. Jour. Theo. Phys. Vol. 1, p. 1, 1953) the differential cross-section has been calculated by the usual methods. The general formula thus derived involves a number of undetermined $\pm$ signs, which are to be so adjusted that the general formula may reduce to Thomson formula obtained by applying the general perturbation

theory to low energy interaction. This method, however, does not completely remove the uncertainty of $\pm$ signs and two alternative formulæ are obtained both of which reduce to Thomson formula for low energy of the incident radiation ($k_0 \leqslant \mu$). Of these one is found to be untenable as it fails to explain large angle scattering at high energy. The other is the exact Klein-Nishina formula.

VOL. 2, NO. 2, 1954

Diffusion of matter in solid Metals and Alloys

G. P. Chatterjee
(B. E. College, Howrah, W. B., India)

ABSTRACT : From Fick's Law of diffusion it may be easily shown that the rate of diffusion $\left(\frac{\delta\mu}{dt}\right)_i$ of the ith component in a system of q components is given by

$$\left(\frac{\delta\mu}{\delta t}\right)_i = \sum_{q=1}^{n} - {}_cD_{iq} \nabla C_q \quad (i,=1, 2, \cdots\cdots\cdots n)$$

where $\left(\frac{\delta\mu}{\delta t}\right)$ is the mass of matter diffusing per unit area in unit time, ${}_cD_{iq}$ represent in general concentration—diffusion co-efficients and ∇C_q is the concentration gradient of the qth component. To overcome the limitations of the above relation different assumptions have been made by different investigators on the basis of different thermodynamic functions like chemical potential, activity and fugacity leading to the equations

$$\frac{\delta\mu}{\delta t} = - {}_\alpha D \frac{\delta\alpha}{\delta x}$$

along some arbitrarily chosen x-axis.

Similarly $$\frac{\delta\mu}{\delta t} = - {}_f D \frac{\delta f}{\delta x}$$

and $$\frac{\delta\mu}{\delta t} = - G_k T \left(1 + \frac{\delta \ln \lambda}{\delta \ln c}\right) \frac{\delta c}{\delta x}$$

where ${}_{a}D$, ${}_{f}D$ are the activity and fugacity diffusion co-efficients, G is the mobility (velocity per unit force), k is the Boltzmann constant, T is the absolute temperature and λ is the activity co-efficient.

An attempt has been made in this paper to develop the concept of diffusion on the basis of potential ϕ and energy density ψ leading to the equation

$$\frac{\delta\mu}{\delta t}=-{}_{\psi}D\phi A\left[\left(\lambda+c\frac{\delta\lambda}{\delta c}\right)+\frac{\lambda c}{\phi}\;\frac{\delta\phi}{\delta c}\right]\frac{\delta c}{\delta x}$$

It has been shown that this generalised equation leads to all the other diffusion equations under centain specified conditions.

VOL. 2, NO. 2, 1954

Relativistic Hamiltonian and Wave equations of an electron in an assembly

S. Sanatani
(Presidency College, Calcutta, India)

ABSTRACT : The method of linearizing the relativistic Hamiltonian of an electron in an assembly without the use of matrices, previously given by Kar and Sanatani, is further discussed here. Linear wave equations have been obtained from the linear Hamiltonian, and it has been shown that even if the spin-orbit interaction energy E_{so} is not included in the Hamiltonian, the correct second order wave equation and the fine structure formula follow automatically.

VOL. 2, NO. 2, 1954

Note on the solution of some problems of semi-infinite elastic solids with transverse isotropy

Bibhuti Bhusan Sen
(Birla College of Science, Rajasthan, India)

ABSTRACT : A simple method of solving problems of semi-infinite elastic solid with transverse isotropy and having symmetrical distribution of shearing stresses on the plane boundary is discussed in this paper.

VOL. 2, NO. 2, 1954

The summation rule for B_p, B_n

Mrs. Amita Basu (Mitra)
(Presidency College, Calcutta, India)

ABSTRACT : A new summation rule for B_p, B_n has been stated. It is shown that with the help of this rule unknown values of B_p, B_n may be determined in many cases.

VOL. 2, NO. 2, 1954

On beta-emission by lighter elements and determination of B_p, B_n

Mrs. Amita Basu (Mitra) and Satyapravat Banerjee
(Presidency College, Calcutta, India)

ABSTRACT : The elements in the region $Z=0$ to $Z=9$ have been studied with respect to binding energies and β-decay phenomenon. The experimentally determined β-decay energies in this region has been checked by different artificial reactions.

In the second part of the paper, almost all the B_p and B_n values in the above mentioned region are determined by means of Q values of artificial reactions. Curves plotted with B_p and B_n values thus determined, against Z or N, exhibit alternate increase and decrease for even and odd values of Z or N.

VOL. 2, NO. 3, 1954

On Bremsstrahlung in the non-relativistic region

R. K. Bhattacharyya
(Surendranath College, Calcutta, India)

ABSTRACT : The differential cross-section for Bremsstrahlung in the non-relativistic region is usually obtained from Bethe-Heitler's relativistic expression by making non-relativistic approximation. In the present paper the non-relativistic

differential cross-section is obtained in a perfectly straight forward manner by using the non-relativistic Hamiltonian for an electron.

VOL. 2, NO. 3, 1954

Nuclear moments and shell structure
(Review paper)

S. Sengupta
(Darjeeling Govt. College, W. B., India)

VOL. 2, NO. 3, 1954

The nature of Yukawa potential

K. C. Kar and Miss Durga Roy
(Presidency College, Calcutta, India)

ABSTRACT : It is shown that the short range Yukawa potential is of electrical origin, being due to polarisation of the nucleons at close distance. It is further shown that for two spinning nucleons at close distance the general interaction energy should be

$$W = -g^2 \frac{e^{-kr}}{r} + \left(\mu_2 \operatorname{rot}_2 \operatorname{rot}_2 \left(\frac{\mu_1}{r} e^{-kr} \right) \right)$$

as taken by Kar and Mukherjee (Ind. Jour. Theo. Phys. Vol. 1, p. 67, 1953 ; Vol. 2, p. 17, 1954).

VOL. 2, NO. 4, 1954

Application of Dirac's δ-function in isolated-force problems of the theory of elasticity

Part I

Ghasi Ram Verma
(Birla College of Science, Rajasthan, India)

ABSTRACT : In this paper problems of isolated forces acting on the straight boundary of a semi-infinite plate, composed of isotropic or non-isotropic material have been solved by using

Dirac's δ-function. The method is found very useful specially when there are a large number of forces acting on the boundary.

VOL. 2, NO. 4, 1954

A resolution condition for X-ray spectral lines

Amar Nath Nigam
(Allahabad University, India)

ABSTRACT : A general condition for resolution of two X-ray lines of unequal intensities and breadths has been deduced.

VOL. 2, NO. 4, 1954

On Badger's rule

Y. P. Varshni and S. S. Mitra
(Allahabad University, India)

ABSTRACT : A semi-theoretical justification of Badger's rule *viz.*, $K_e(r_e-d)^3=C$ has been given on a classical model of diatomic molecule. The constant C of a molecule PQ has been shown to be related with the similar constants of molecules PP and QQ.

VOL. 2, NO. 4 1954

Note on the nature of (dp) reaction

Mrs. A. Basu (Mitra)
(Presidency College, Calcutta, India)

ABSTRACT : It is shown that though the Q values of excited states in case of (dn) reactions give several B_p values, the B_n values are not actually found from the corresponding Q values in (dp) reactions, since in the latter case the proton levels are excited.

VOL 2, NO. 4, 1954

VOLUME 3

1955

The experimental study of Bowed string and verification of the formula for the bowing pressure

R. N. Bagchi and Mrs. Chinmayee Dutta
(Presidency College, Calcutta, India)

ABSTRACT : The paper deals with a quantitative verification of Kar formula of bowed string, *viz.*,

$$P_{min} = \alpha\mu_D \left\{ \frac{l}{d} (V - V_o) + V_o \right\} + P_o$$

where P_{min} is minimum bowing pressure to excite a steady vibration of the string; μ_D is the dynamical co-efficient of friction during backward motion of the string, α a constant of proportion, l the length of the vibrating string, d the bowing distance, V the bowing velocity, $V - V_o$ the velocity of the string at the mid-point of the bowed region and P_o the additional pressure term due to resonance, statical displacement and other factors. To determine V_o of the above formula the variation of exact position of the zero-point with different distances and velocities are thoroughly studied by photographic method. From a set af values of P_{min} at different bowing distances for a given frequency and bowing velocity $\alpha\mu_D$ and P_o are calculated. These calculations have been repeated for two different velocities and frequencies.

VOL. 3, NO. 1, 1955

Torsional vibration of a cylinder of cylindrically aeolotropic material

J. G. Chakravorty
(Bangabasi College, Calcutta, India)

ABSTRACT : The problem of torsional vibration of a cylinder of a material with cylindrical aeolotropy about the axis of the

cylinder is discussed in this paper. An exact solution of the problem has been obtained when the cylinder is either hollow or has an isotropic core.

VOL. 3, NO. 1, 1955

Wavestatistical derivation of the formula for transition probability

S. Sanatani
(Presidency College, Calcutta, India)

ABSTRACT : The well known formula for transition probability for direct transitions or transitions through intermediate states, is derived by a new method involving χ_1 and χ_2 waves in phase space.

VOL. 3, NO. 1, 1955

On the variation of binding energy of the last nucleon and the nuclear exchange forces

S. Ghosh and S. Sengupta
(Darjeeling Govt. College, W. B., India)

ABSTRACT : It is shown in the following that the assumptions made regarding B_{NN} etc. by Sengupta (1953) in an earlier paper are derivable from a mixture of Wigner and Majorana forces. Comparison with experimental values of B_{NN} etc. are also made. A new characteristic is revealed in the variation of $B_{NZ}(N, Z)$ with the parity of $N+Z$, which is interpreted as showing the existence of a spin dependent term in the nuclear potential.

VOL. 3, NO. 1, 1955

On the summation rule for B_p and B_n, Pt. I

K. C. Kar and Mrs. Amita Basu
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The validity of the summation rule for B_p, B_n (*vide*, Ind. Jour. Theo. Phys. Vol. 2, No. 2, 1954) has been

proved up to the atomic number $Z=30$. Using this rule some doubtful B_p and B_n values have been corrected and several unknown B_p's and B_n's have been calculated in this region.

VOL. 3, NO. 2, 1955

Dynamics of the vibration of a bar excited by Transverse impact of a hard load (Part I)

M. Ghosh and K. D. Ray
(City College, Calcutta, India)

ABSTRACT : Dynamics of the vibration of a bar fixed at one end and free at other end, excited by *transverse impact* by a *hard load* at the free end has been worked out, following operational method. The main idea upon which the dynamics is built up is, that the bar behaves like a *loaded bar* so long as the load is in contact with it. The expression for the displacement at struck point, pressure exerted by the load during impact, duration of impact and relative energy communicated to the bar by the impinging load have been worked out. The variation of relative energy with the change of the mass-ratio of the bar and the load has been experimentally observed. It is found that the theory built up is nicely supported by observations.

VOL. 3, NO. 2, 1955

The cross-section theorem in quantum mechanics

David Park
(University of Ceylon, Colombo)

ABSTRACT : This paper gives the most general statement and proof of the relation between total cross section and forward scattering amplitude. The relation is shown to hold regardless of the nature of the transitions which occur, provided only that the entire system can be described by a normalizable state function. Examples are given showing how to find the explicit formula in situations where exchange is involved.

VOL. 3, NO. 2, 1955

The theory of alpha disintegration

K. C. Kar and S. P. Banerjee
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : It is shown that the upper limit of s introduced in the theory of Kar and Kundu (1953) should not be 2. It should be determined for a given Z from the experimental value of r'_0 the lower limit of Coulomb scattering. With the value of s thus determined from Farwell and Wegner's (Phys. Rev. Vol. 95 p. 1212, 1954) data for $Z=82$ and $Z=88$, namely, $s=6{\cdot}184$ and $s=10{\cdot}350$ respectively, it is found that the theoretical values of disintegration constants of isotopes having no finestructure agree quite well with the experimental values and lie on a smooth $\log \lambda - E$ curve. The theoretical and experimental values of $\log \lambda$ for minimum energy of emission in case of finestructure also lie on this curve. The agreement is decidedly better than with Gamow's theory. It is further shown that the general downward trends of $\log \lambda - E$ curves in case of fine-structure are also satisfactorily explained in the new theory. Gamow's theory completely fails to explain this characteristic of finestructure.

VOL. 3, NO. 3, 1955

Stresses in a rotating blade bounded by two equal confocal parabolas

B. B. Chatterjee
(College of Engineering & Technology, Jadavpur, Calcutta, India)

ABSTRACT : In this paper the stresses in blades bounded by two equal confocal parabolas rotating about perpendicular axes through the centre and one of the points of intersection of the parabolas respectively have been discussed.

VOL. 3, NO. 3, 1955

Two-dimensional problems of stress distribution due to certain loads on the upper surface of an elastic layer of non-isotropic material with rigid base

Pritindu Choudhury
(Krishnagar College, Nadia, W. B., India)

ABSTRACT : The paper is concerned with the determination of two-dimensional stresses in a layer of aeolotropic material resting on a rigid base, the upper surface having various distributions of normal loads. Problems have been solved by using Fourier transform.

VOL. 3, NO. 3, 1955

On the stress distribution in an infinite elastic solid of a transversely isotropic material with a cylindrical hole due to a localised axial shear

J. G. Chakravorty
(Bangabasi College, Calcutta, India)

ABSTRACT : The problem of localised axial shear on the boundary of a cylindrical hole in an infinite solid of a transversely isotropic material has been considerd in this paper.

VOL. 3, NO. 3, 1955

On the summation rule for B_p and B_n Pt. II

K. C. Kar and Mrs. Amita Basu
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The paper deals with the calculation of nearly twenty unknown values of B_p and B_n in the region : $Z=31$ to $Z=38$, with the help of the summation rule.

VOL. 3, NO. 4, 1955

Operator methods in classical field theory

David Park
(University of Ceylon, Colombo)

ABSTRACT : It is shown how the solution of certain classical problems in the theory of fields—Poisson's integral and its Fourier transform, Lagrange's expansion, the Lienard-Wiechart potentials, Cauchy's problem etc.—can be treated in a simple and unified way by the use of operator methods, in which algebraic and analytic processes are suitably combined. The methods developed in the foregoing are then used to discuss an equation satisfied by retarded potentials only which is of the first order in the time derivative.

VOL. 3, NO. 4, 1955

Dynamics of the vibration of a bar excited by transverse impact of an elastic load (Part II)

M. Ghosh and K. Ray
(City College, Calcutta, India)

ABSTRACT : This paper deals with dynamics of vibration of the bar fixed at one end struck transversely by an elastic hammer at the free end. The bar is assumed to behave as a loaded bar so long as the hammer is in contact with it and the elastic hammer is conceived of as a perfectly hard hammer backed by a weightless spring. The expressions for the displacement at struck point, pressure exerted by the load during impact, have been worked out. The case of a hard hammer has been deduced as a special case by putting the elastic constant of the hammer equal to infinity. The results obtained in part I come also as special cases.

VOL. 3, NO. 4, 1955

VOLUME 4

1956

On the summation rule for B_p and B_n, Pt. III

Mrs. A Basu
(Presidency College, Calcutta, India)

ABSTRACT : The summation rule for B_p, B_n has been used to calculate the unknown binding energies of last protons and last neutrons for the atomic numbers betweeen $Z=39$ to $Z=48$.

VOL. 4, NO. 1, 1956

On an extension of Kar formula of minimum bowing pressure

R. N. Bagchi
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : Kar formula for minimum bowing pressure (Ind. Jour. Theo. Phys. 2, p. 46, 1954) is extended by calculating the frictional force more rigorously. The formula thus obtained satisfactorily explains both the straight portion (at high velocities) and the upward bend (at low velocities) of the $P_{min}-V$ curve.

VOL. 4, NO. 1, 1956

On the nuclear shell structure

K. C. Kar and S. P. Banerjee
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The eigenfunctions and discrete values of energy of nucleons within a nucleus have been determined by a rigorous method for nuclear potential $U=-U_o+\frac{1}{2}\mu r^2$. It is shown that when μ is large the eigenfunctions are same as

those deduced by Bethe and Bacher by their round-about method. When, however, μ is small the eigenfunctions are same as those for constant potential with some correction factors which reduce to unity when $\mu \rightarrow 0$.

VOL. 4, NO. 1, 1956

Nuclear electric and magnetic moment in j-j coupling

S. Sengupta
(Darjeeling Govt. College, W. B., India)

ABSTRACT : With the help of the diagonal sum rule nuclear moment in j-j coupling can be calculated without explicitly setting up the corresponding wave functions. Sengupta (1950) previously applied the principle to determine the multiplet separation factors in atomic spectra. Lawson (1955) used the same principle to calculate the nuclear electric quadrupole moment for equivalent proton configurations. In the present paper this method has been generalised. Closed expressions are given for the magnetic dipole moment and electric quadrupole moment in nuclear configurations containing both neutron and proton and belonging to different values of l. Application of the equations is also considered in two cases.

VOL. 4, NO. 2, 1956

Longitudinal propagation of elastic disturbance for linear variations of elastic parameters

A. N. Datta
(B. E. College, Howrah, W. B., India)

ABSTRACT : One dimensional propagation of elastic disturbance in an isotropic medium for linear variations of elastic parameters has been considered. The problem has been solved for three different cases : for a source having (1) periodic displacement, (2) periodic stress, and (3) impulsive stress. For the solution of (3) Dirac's δ-function has been used. The solutions have been obtained in Bessel fnnctions of zero order. Further simplifications have been obtained for small variations of the parameters using asymptotic expansions for Bessel functions.

VOL. 4, NO. 2, 1956

The statical displacement in Bowed string

Mrs. Chinmayee Dutta and Sunil Baran Niyogi
(Presidency College, Calcutta, India)

ABSTRACT : The paper deals with the experimental investigation of the problem of the statical displacement which may be defined as the displacement of the mean position of the string during vibration, from its undisturbed position. This statical displacement evidently causes asymmetry in the vibration. It is found that nearabout the bowed region the string vibrates asymmetrically and that the asymmetry depends on the bowing distance and the frequency of vibration of the string.

VOL. 4, NO. 2, 1956

Method of conjugate functions applied to a problem in electricity

Narendra Lal Maria
(Govt. Technical Institute, Ambala)

VOL. 4, NO. 2, 1956

Twisting of a hollow circular cylinder of cylindrically aelotropic material with outer surface fixed and inner surface acted on by tangential tractions

P. P. Chatterji
(Calcutta Technical School, Calcutta, India)

ABSTRACT : Axially symmetric deformations and torsional stresses in a hollow circular cylinder of cylindrically aelotropic material with outer surface fixed and inner surface acted on by tangential tractions have been determined. The case when the tractions are distributed uniformly over portions of the inner boundary is also considered.

VOL. 4, NO. 3, 1956

Dynamics of the vibration of a bar excited by transverse impact of an elastic load : displacement at any point during impact

K. D. Ray
(City College, Calcutta, India)

ABSTRACT : This paper deals with the displacement at any point during impact of a cantilever struck transversely by any load at any point $x=a$. The idea of the previous paper is retained and the portion of the bar beyond the struck point is assumed to remain straight. The other special cases have also been worked out.

VOL. 4, NO. 3, 1956

The intensities in fine structures in X-ray absorption spectra of ionic solutions

Amar Nath Nigam
(Allahabad University, Allahabad, India)

ABSTRACT : It has been shown that in order to obtain theoretically the ratio of intensities in the K-edge finestructures of Zn^{++} ion in an ionic solution, it is essential to consider the influence of the Debye-Hückel cloud. Transition probabilities with the modified wave function of the final state have been calculated.

VOL. 4, NO. 3, 1956

Determination of B_p, B_n by graphical method

S. P. Banerjee
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : Thirteen new values of B_p and B_n have been obtained by graphical method. By chosing suitable parameters in each case, smooth curves for B_p and B_n have been obtained. Some of the unknown values have been directly interpolated or extrapolated from the curves, while the rest have been obtained by using these values and other values known from the sum-rule.

VOL. 4, NO. 4, 1956

Note on a type of distortionless transmission line with variable line parameters

Bibhuti Bhusan Sen
(Jadavpur University, Calcutta, India)

ABSTRACT: Condition for the propagation of electric disturbances without distortion in a certain type of non-uniform transmission line has been discussed in this note.

VOL. 4, NO. 4, 1956

On the stresses in a composite truncated cone due to shearing stresses on the curved surface

Sisir Chandra Das
(Chandernagore College, W. B., India)

ABSTRACT: Solutions have been obtained for the problem of a truncated cone with the same axis but of a different material when the outer curved surface is under uniform shearing stresses. The particular type of applied stress discussed in this paper develops due to frictional forces that cause the twisting of the solid about its axis.

VOL. 4, NO. 4, 1956

Stresses in a circular cylinder and in a paraboloid of revolution due to shearing forces produced by circular rings of the curved surface

G. R. Verma
(Birla College of Science, Rajasthan, India)

ABSTRACT: Using Dirac's delta function, problems of a circular cylinder and a paraboloid of revolution twisted by tightly fitting rings on the curved surface have been solved in this paper.

VOL. 4, NO. 4, 1956

VOLUME 5

1957

Determination of B_p, B_n in the region : $Z=49$ to $Z=54$

Amita Basu
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : A large number of unknown B_p and B_n values have been calculated by sum rule in the region $Z=49$ to $Z=54$. In some cases, these calculated values have been checked as well as some new B_p, B_n values have been calculated by beta decay energy, position decay energy, or artificial reaction energy, in the above mentioned region.

VOL. 5, NO. 1, 19'7

Stresses in an aeolotropic paraboloid of revolution due to frictional force acting on its surface

S. B. Dutt
(Calcutta Technical School, Calcutta, India)

ABSTRACT : The present paper aims at finding the displacement in a paraboloid of revolution with a material having curvilinear aeolotropy. The shearing stresses which are of the nature of frictional forces and are responsible for the twisting of the solid about its axis of revolution are supposed to be uniformly distributed over the curved surface of the solid of revolution.

VOL. 5, NO. 1, 1957

Stresses in a spherically isotropic truncated cone fitted with two rigid spherical caps due to frictional forces acting on the curved surface

P. P. Chattarji
(Calcutta Technical School, Calcutta, India)

ABSTRACT : Torsional stresses and deformations in a spherically isotropic truncated cone fitted with two rigid spherical caps due to frictional forces acting on the curved surface have been obtained.

VOL. 5, NO. 1, 1957

Note on the dynamical co-efficient of friction

K. C. Kar and Mrs. Chinmayee Dutta
(Institute of Theoretical Physics, Calcutta, India)

VOL. 5, NO. 1, 1957

The role played by modulus of rigidity in the process of fusion

P. K. Chatterjee
(B. E. College, Howrah, W. B., India)

ABSTRACT : A new aspect of the process of fusion is presented and a quantitative theory relating the latent heat of fusion of a substance with its modulus of rigidity is developed from very elementary considerations. The theoretical formula is then tested by applying it to a number of elements and the validity and the merit of the theory are discussed on the basis of the results of these calculations.

VOL. 5, NO. 2, 1957

Extension of Kar formula for minimum bowing pressure

K. C. Kar and Mrs. Chinmayee Dutta
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : Kar formula for minimum bowing pressure (Ind. Jour. Theo. Phys. Vol. 2, p. 47, 1954) is extended for low velocity of bowing. The formula thus obtained satisfactorily explains both the straight portion at high velocities and the upward bend at low velocities of $P_{min}-V$ curve. It is also shown that the frictional forces due to bowing are same during forward and backward motion of the string.

VOL. 5, NO. 2, 1957

On the solution of transport equation for neutron diffusion in the Milne-Eddington model

Santi Ranjan Das Gupta
(City College, Calcutta, India)

ABSTRACT : We consider the simple transport equation governing the passage of mono-energetic neutrons through a large slab of non-capturing medium having a uniformly distributed neutron sources of a constant strength. The neutrons are scattered anisotropically without change of energy. Apart from the current contributed by the neutron sources, a constant steady neutron current is otherwise maintained in the outward direction normal to the plane face of the slab. The emergent neutron distribution $I(0, \mu)$ is given in a closed form as an explicit function of $\mu(=\cos\theta)$ defined for the entire complex μ-plane. When scattering is not highly anisotropic, if in the expansion of the scattering cross-section in spherical harmonics we retain the first two terms it is seen that the neutrons after being scattered anisotropically while diffusing through the non-capturing medium, show complete isotropy after emergence. If, however, in the above expansion the first three terms are retained anisotropy arising out of the second term is absent in

the emergent beam which only shows non-isotropy to the extent measured by the third term. When the effect of the third term is taken to be very small a simple closed form of $I(0, \mu)$ can be quite easily obtained.

VOL. 5, NO. 2, 1957

A new derivation of Klein-Nishina formula without matrices

K. C. Kar and B. N. Paria
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : Klein-Nishina formula for differential cross-section of Compton scattering, deduced so far only on the basis of Dirac's theory of electron, is derived here by an alternative method without using Dirac's non-commutative matrices and hences avoiding lengthy and intricate "spur" calculation. The exact formula for the differential cross-section has been uniquely obtained in a perfectly simple and straightforward manner. In so doing, the linear form of Hamiltonian recently obtained by Kar and Sanatani (Ind. Jour. Theo. Phys. Vol. 1, p. 1, 1953) and the ordinary multiplication rules of vectors have been used. The physical significances of the different mathematical steps are also clearly explained.

VOL. 5, NO. 3, 1957

Graphical determination of B_p, B_n (Part II)

S. P. Banerjee
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : Fourteen new values of B_p, B_n have been found out from a graphical study of binding energies of last neutron and proton in nuclei. This is a continuation of a previous paper by the author where smooth curves for B_p, B_n were obtained by choosing suitable parameters. The smoothness has also been shown to be exhibited here. The range selected is from $Z=19$ to $Z=25$ and $N=20$ to $N=26$.

VOL. 5, NO. 3, 1957

Note on some problems of thin equilateral triangular plate

Bibhutibhusan Sen
(Jadavpur University, Calcutta, India)

ABSTRACT : Trilinear co-ordinates are used to solve some problems of vibrations of a thin equilateral triangular plate with supported edges.

VOL. 5, NO. 3, 1957

A simple theory of Bremsstrahlung without Dirac matrices

K. C. Kar and B. N. Paria
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In the present paper, the relativistic differential cross-section for Bremsstrahlung has been deduced in a perfectly simple and straight-forward manner without using Dirac's non-commutative matrices and hence avoiding lengthy and tedious 'spur' calculations. The linear form of Hamiltonian as given by Kar and Sanatani (Ind. Jour. Theo. Phys. Vol. 1, No. 1, 1953) and the simple multiplication rules of vectors have been used.

VOL. 5, NO. 4, 1957

On bending of a circular plate of aeolotropic material under certain non-uniform distribution of load

P. Choudhury
(Krishnagar College, W. B., India)

ABSTRACT : In this paper, solution of the fundamental equation for an elastic material with transverse isotropy has been obtained in the form of polynomials and it has been used in discussing the problem of bending of a circular plate under non-uniformly distributed load, the intensity of which following a parabolic law of distribution.

VOL. 5, NO. 4, 1957

On the verification of the extended Kar formula for minimum bowing pressure

Mrs. Chinmayee Dutta
(Presidency College, Calcutta, India)

ABSTRACT : The extended Kar formula for minimum bowing pressure is verified by the data obtained by the writer in a previous experiment. (*vide* : Ind. Jour. Theo. Phys. Vol. 3, p. 5, 1955)

VOL. 5, NO. 4, 1957

On the smoothness of B_p, B_n graphs

Satyaprabhat Banerjee
(Institute of Theoretical Physics, Calcutta, India)

VOL. 5, NO. 4, 1957

On the process of β-emmission

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

VOL. 5, NO. 4, 1957

VOLUME 6

1958

Energy levels in a bounded isotropic harmonic oscillator potential and nuclear shell structure

S. Sengupta and S. Ghosh
(Darjeeling Govt. College, W. B., India)

ABSTRACT : Quantum mechanical problem of a nucleon moving within an isotropic harmonic oscillator potential of value $V=\frac{1}{2}kr^2$, and enclosed within a spherical box with rigid walls

(infinite potential well) is investigated. Approximate expressions for energy levels are obtained. These levels are essentially determined by the parameter $\rho = \dfrac{\hbar\omega}{\hbar^2/MR^2}$ where $\omega = \sqrt{\dfrac{k}{M}}$ is the classical circular frequency of the oscillator and R is the radius of the box. For small ρ energy levels tend to those of infinite potential well and for large ρ they tend to the energy levels of an unbounded oscillator. Introducing a spin orbit coupling term 30 times Thomas value and for $\rho = 6$ and $R = 1{\cdot}3 \times 10^{-13} A^{\frac{1}{3}}$ excellent agreement with experimental shell structure is obtained. Level schemes are in close agreement with the observed ones.

VOL. 6, NO. 1, 1958

On Femi theory and Sargent rule for β-decay

Sabita Purkayastha
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In this paper Sargent's rule has been studied from the standpoint of Fermi's theory. A number of theoretical Sargent curves have been drawn with different constants according to Fermi's formula. The experimental points are found to lie on these curves even in the bent portion at low energy region irrespective of the atomic number Z and $\triangle I$.

VOL. 6. NO. 1, 1958

Note on the bending of a thin uniformly loaded plate bounded by two equal confocal parabolas

B. B. Chatterjee
(Jadavpur University, Calcutta, India)

ABSTRACT : In this note the bending of a thin uniformly loaded plate bounded by two equal confocal parabolas is discussed. The boundary is assumped to be clamped and the plate, bent by transverse forces.

VOL. 6, NO. 1, 1958

On the exact solution of transfer equation in the Milne-Eddington model : Conservative, grey isotropic scattering

Santi Ranjan Das Gupta
(City College, Calcutta, India)

ABSTRACT : In the present paper we are considering the conservative case of isotropic grey scattering of radiation in the outer layers of a star. Chandrasekher's H-function $H(\mu)$ (giving the law of limb darkening) is obtained in an alternative integral form which can be tackled much more easily than the integral form of the same deduced by Chandrasekhar[2]. A definite advantage of the present integral form lies in the fact that from it we can deduce an analytically consistent closed form of $H(\mu)$ defined for the entire complex μ-plane, by a polynomial curve-fitting in the range (0, 1). There arise certain problems wherein a closed form of $H(\mu)$ (defined for the entire μ-plane) proves to be of valuable utility in mathematical analysis as well as in minimising labour of numerical computations. The present method of solution differs from the usual Wiener-Hopf technique only in the initial set-up which is so designed as to get the final integral with limits of integration between 0 and 1 instead of between $-\infty$ and ∞. If high accuracy is not wanted we can have a closed form of $H(\mu)$ involving a curve-fitting polynomial of degree 4. Values of $H(\mu)$ obtained from this closed form agree up to third place after the decimal point, with Chandrasekhar's determinations of the same from the numerical evaluation of his integral form of $H(\mu)$.

VOL. 6, NO. 2, 1958

Stress concentrations around a small inclusion on the axis of a circular cylinder under torsion

P. P. Chattarji
(Calcutta Technical School, Calcutta, India)

ABSTRACT : Stresses are calculated for the case of a small rigid inclusion (*i. e.*, zero displacement on the surface of the inclusion) bonded to a large circular cylindrical shaft in torsion,

the axis of the inclusion coinciding with that of the cylinder. The inclusions considered are of the shapes (*i*) two spheres not touching each other, (*ii*) two spheres touching each other; (*iii*) inverse of an ovary ellipsoid with respect to its centre.

VOL. 6, NO. 2, 1958

On linearisation of the relativistic Hamiltonian

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT: It is shown that the relativistic Hamiltonian for a free electron in an assembly is a vector and so can be linearised directly without introducing unnecessary complications of Dirac matrices.

VOL. 6, NO. 2, 1958

Pair production on the basis of hole theory without Dirac matrices

Birendra Nath Paria
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT: The differential cross-section for the creation of a pair of electron and positron by γ-ray in the presence of a nucleus has been deduced in a simple and straightforward manner taking Dirac's idea of 'hole' but without using his matrices. This new method is a general one and can be applied successfully to all radiation problems.

VOL. 6, NO. 3, 1958

On the exact solution of transfer equation in the Milne-Eddington model: Scattering according to Rayleigh phase function

Santi Ranjan Das Gupta
(City College, Calcutta, India)

ABSTRACT: The transfer equation for the conservative case of grey scattering according to Rayleigh phase function is exactly

solved by Wiener-Hopf technique. The law of darkening $H(z)$ is obtained in a closed form, as an explicit function of z, defined for the entire complex z-plane. The asymptotic form of the average intensity comes out as a linear function of the optical depth.

VOL. 6, NO. 3, 1958

On disturbances generated by a pulse of pressure on the surface of a spherical cavity in an elastic medium

Sakti Kanta Chakraborty
(Jadavpur University, Calcutta, India)

ABSTRACT : The propagation of disturbances in an elastic infinite medium has been considered when the disturbance is produced by a sinusoidal pulse of pressure applied on the surface of a spherical cavity in the medium.

VOL. 6, NO. 3, 1958

Note on the torsion of a curved rod of circular cross section with transverse isotropy

S. B. Dutt
(Calcutta Technical School, Calcutta, India)

ABSTRACT : In this paper the problem of the torsion of a curved rod of circular cross-section with transverse isotropy has been investigated. An exact solution has been obtained by introducing a single stress function satisfying a partial differential equation which has been solved with the use of toroidal co-ordinates.

VOL. 6, NO. 4, 1958

Derivation of the wave functions for $j=1+\frac{1}{2}$, $j=1-\frac{1}{2}$ states and for '0' spin using Kar's linear Hamiltonian

Sabita Purkayastha
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In this paper the exact wave functions representing the two components of the energy levels due to spin, have been obtained in a straightforward way, directly from the second order R-equation. The wave equation of a particle with zero spin has also been deduced starting from Kar's linear Hamiltonian.

VOL. 6, NO. 4, 1958

Note on the linearisation of the relativistic Hamiltonian

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The relativistic Hamiltonian has been linearised by a perfectly logical wavestatistical method without introducing Dirac matrices at any stage.

VOL. 6, NO. 4, 1958

VOLUME 7

1959

Note on the forced torsional vibration of a cylinder having periodic shearing forces along a ring on the curved surface

A. K. Mitra
(Jadavpur University, Calcutta, India)

ABSTRACT : This brief note deals with the investigation of the forced vibration of a cylinder as a result of a shearing

stress applied along the circumference of a circle on the surface of the cylinder at a certain height from the lower end of the cylinder which is fixed. Solutions have been obtained (*i*) when both ends are fixed and (*ii*) when one end is fixed the other end being free.

VOL. 7, NO. 1, 1959

On the work done during bowing

Mrs. Chinmayee Dutta
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The work done by the bow during forward motion of the string and also the work done on the bow during its backward motion are separately calculated for any velocity of bowing. As the works done in the two cases are equal for maintained vibration it is shown that the bowing pressure is constant during forward and backward motion of the string.

VOL. 7, NO. 1, 1959

Torsion and bending of an aeolotropic beam having a curtate section

A. Chakravorti
(Brahmananda Keshab Chandra College,
Calcutta, India)

ABSTRACT : Following the semi-inverse method of St. Venant components of stress associated with torsion and bending of an aeolotropic beam are obtained in this paper. The cross section of the beam considered is a curvilinear rectangle bounded by two concentric circles and two radii.

VOL. 7, NO. 1, 1959

Linear relativistic Hamiltonian and the electromagnetic field

K. C. Kar and Sabita Purkayastha
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT: The direct method and the method of taking wavestatistical average, for the linearisation of the relativistic Hamiltonian, without matrices have been critically discussed. It is shown that the relativistic Hamiltonian is a world vector in the space-time continuum. The linear Hamiltonian without matrices for charged particle in the electromagnetic field has also been studied. It is shown that the wellknown term involving H representing the energy of a magnetic dipole comes out on recombining the two linear Hamiltonian in the electromagnetic field.

VOL. 7, NO. 2, 1959

On the relativity of uniform rotation

Anadi J. Das
(Calcutta University, Calcutta, India)

ABSTRACT: The meaning of uniform rotation is properly analysed and clarified in the light of the principle of relativity. It is shown that uniform rotation is not an absolute motion and there exists a special relativity of uniform rotation.

VOL. 7, NO. 2, 1959

Note on the uniqueness of solution of problems connected with thin plates bent by normal pressures

Bibhutibhusan Sen
(Jadavpur University, Calcutta, India)

ABSTRACT: The object of this note is to prove that the solution of the problem of a thin plate which is in a state of uniform tension, and is bent by normal pressures, is unique for different edge conditions.

VOL. 7, NO. 2, 1959

On the estimate of the asymptotic value of optical thickness of a spherically symmetric stellar atmosphere in the non-conservation case in the second approximation

K. K. Sen
(University of Malaya, Singapore)

ABSTRACT: The asymptotic value of the optical thickness of a spherically symmetric stellar atmosphere has been calculated in the second approximation for non-conservative isotropic scattering case. Wick-Chandrasekhar method has been used for solving the integro-differential equation of transfer appropriate to the problem.

VOL. 7, NOS. 3 & 4, 1959

Note on Dirac method of linearising the relativistic Hamiltonian

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT: It is pointed out that conditions for linearising the relativistic Hamiltonian by Dirac method are ignored in its subsequent use. It is also shown that all the Dirac results can be obtained if unit vectors are introduced instead of unnecessarily complicated matrices.

VOL. 7, NOS. 3 & 4, 1959

Note on the wavestatistical derivation of Klein-Nishina formula for compton scattering

K. C. Kar and B. N. Paria
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT: The wave-statistical theory of Klein-Nishina formula and the mechanisms of the different processes involved have been critically discussed.

VOL. 7, NOS. 3 & 4, 1959

Note on twisting of an isotropic circular cylinder embedded in a resisting medium acted upon by couple at one end, the other end being fixed

A. K. Das
(Jadavpur Polytechnic, Calcutta, India)

ABSTRACT : The object of this note is to solve the problem of an isotropic circular cylinder embedded in a resisting medium when a twisting couple acts at one end, the other end being fixed.

VOL. 7, NOS. 3 & 4, 1959

On the perturbation theory with time-dependent wave functions

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The wellknown perturbation formula used in radiation problems has been derived from an entirely wave statistical point of view.

VOL. 7, NOS. 3 & 4, 1959

VOLUME 8

1960

Dynamics of the vibration of bar excited by transverse impact of a load (Experimental)

K. Ray
(Suri Vidyasagar College, Birbhum, W. B., India)

ABSTRACT : The paper deals with the experimental arrangement and study of (*i*) duration of impact, when a particular hammer strikes the free ends of bars of different

lengths or when different hammers strike a particular bar at the free end ; (*ii*) maximum amplitude of vibration during impact when different hammers strike the free end of a bar and (*iii*) relative loss of energy of the hammer due to impact with bars of different mass-ratios. The theoretical expressions for the above quantities have been developed in a series of papers. The special case of a hard hammer of comparatively heavy mass-ratio has been discussed. The agreement between the theory and the experiment is found to be excellent.

VOL. 8, NO. 1, 1960

Note on the displacements in a heavy vertical circular disk with fixed edge and having an isolated load at a point inside the disk

Smriti Kana Bhowmick
(Jadavpur University, Calcutta, India)

ABSTRACT : The object of this note is to solve directly the problem of a vertically placed heavy circular disk which carries a load and has its edge fixed.

VOL. 8, NO. 1, 1960

Note on the torsional vibration of a thin beam of varying cross-section

A. K. Mitra
(Jadavpur University, Calcutta, India)

ABSTRACT : In this note the problem of torsional vibration of a beam of varying cross-section has been considered. In order that the frequency equation can take a simple form, we assume the *moment of inertia* and the *torsional rigidity* to be proportional to e^{4ks} where s is the span-wise co-ordinate and k is a real number. This assumption holds for an exponentially tapering bar.

VOL. 8, NO. 1, 1960

On the theory of 'statical displacement' of a bowed string

Mrs. Chinmayee Dutta
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The present paper gives a simple theory of the statical displacement, which may be defined as the displacement of the mean position of the string during vibration, from its undisturbed position. It is found that the theory is in good agreement with the experimental values previously obtained by the writer.

VOL. 8, NOS. 2 & 3, 1960

The elasto-plastic problem of a thin rotating disc with a central circular hole and variable thickness : Part I

Dipti Dev
(Jadavpur University, Calcutta, India)

ABSTRACT : The purpose of this article is to investigate the problem of a thin rotating disc with a central circular hole and thickness varying with radial distance r as $h=h_o\, e^{-kr^2}$. The problem has been solved in the elasto-plastic range and the angular velocity when the inner boundary attains plastic condition has been calculated.

VOL. 8, NOS. 2 & 3, 1960

Note on the slow steady rotation of a spindle in a viscous fluid

S. R. Majumder

(Jadavpur University, Calcutta, India)

ABSTRACT : In this note the problem of slow steady rotation of a spindle in a viscous fluid has been considered.

VOL. 8, NOS. 2 & 3, 1960

Theory of alpha disintegration

K. C. Kar and A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In the present paper Kar and Kundu theory (Ind. Jour. Theo. Phys. Vol. 1, p. 87, 1953) has been extended by introducting for the first time the idea of probability of formation of alpha particle within the nucleus. A more or less complete explanation of non-fine structure and fine structure in spontaneous alpha disintegration has been given.

VOL. 8, NOS. 2 & 3, 1960

Note on stresses in a truncated cone with the ends fixed due to shearing forces produced by a circular ring on the curved surface

A. K. Das
(Jadavpur Polytechnic, Calcutta, India)

ABSTRACT : The object of this note is to find the stresses in a truncated cone with the ends fixed due to shearing forces produced by a circular ring on the curved surface.

VOL. 8, NO. 4, 1960

Note on the effect of a rigid inclusion in the form of an ovaloid in an infinite plate under all round tension or shear

Miss Smriti Kana Bhowmick
(Jadavpur University, Calcutta, India)

ABSTRACT : The effect of a rigid inclusion in the form of an ovaloid in an infinite plate under all-round tension or shear has been discussed in this paper.

VOL. 8, NO. 4, 1960

Note on stresses in a thin inhomogeneous circular plate with central circular hole, rotating about a diameter

Dipti Deb
(Jadavpur University, Calcutta, India)

ABSTRACT : In this note the problem of a non-homogeneous circular plate rotating about a diameter has been discussed.

VOL. 8, NO. 4, 1960

VOLUME 9

1961

Bending of a cantilever of piezoelectric material loaded at one end

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : The paper is concerned with the piezoelectric analogue of the elastic problem of bending of a cantilever loaded at the end.

VOL. 9, NO. 1, 1961

Fallacies in Dirac method of linearising the relativistic Hamiltonian

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : It is shown that Dirac's method of linearising the relativistic Hamiltonian is open to many objections. Inspite of these objections correct results are obtained by convenient selection of matrices.

VOL. 9, NO. 1, 1961

Note on the stresses in a twisted circular cylinder with modulus of rigidity varying exponentially

Tara Ghosh

(Basanti Debi College, Calcutta, India)

ABSTRACT : In this note the stresses and the displacement in a twisted circular cylinder with modulus of rigidity varying exponentially as $\mu = \mu_0 e^{\mu_1 z}$, where μ_0 and μ_1 are constants and z is the distance from the fixed end have been considered.

VOL. 9, NO. 1, 1961

On the Debye temperature of crystals

S. K. Joshi and B. M. S. Kashyap

(Allahabad University, Allahabad, India)

ABSTRACT : The Debye characteristic temperature is evaluated for a number of crystals belonging to trigonal, tetragonal and orthorhombic systems following the method of Post. The results thus obtained are compared with those deduced by Houston's method. This shows that Post's method does not yield very reliable values for crystals of low symmetry.

VOL. 9, NOS. 2 & 3, 1961

Wavestatistics and the theory of wave field

K. C. Kar and Sabita Purkayastha

(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In this paper the wave functions χ_1, χ_2 have been used as generalised co-ordinates and justification for using them as such has been discussed from the wave statistical standpoint. With these co-ordinates the Lagrangians and Hamiltonians have been constructed and the generalised momenta π_1, π_2 corresponding to χ_1, χ_2 have also been derived. The generalised phase spaces and the generalised wave functions ψ_1, ψ_2 dependent on χ_1, χ_2 have been conceived. When, however, these wave functions are projected in the q-space they are found to behave as χ_1 and χ_2.

VOL. 9, NOS. 2 & 3, 1961

Note on the symmetrical bending of non-homogeneous cylindrical shell of variable thickness

Sukumar Basuli
(B. E. College, Howrah, W. B., India)

ABSTRACT : Radial deflection of a non-homogeneous cylindrical shell of variable thickness has been obtained in this note. The thickness at a section being proportional to the square of its distance from the origin whereas the Young's modulus varies as any power of the distance.

VOL. 9, NOS. 2 & 3, 1961

Note on twisting of non-homogeneous truncated cone

A. K. Das
(S. D. B. College, Howrah, W. B., India)

ABSTRACT : In this brief note twisting of truncated cone, when the material is non-homogeneous, by constant twisting couple applied on the curved surface of the cone has been considered.

VOL. 9, NOS. 2 & 3, 1961

Note on thermoelastic stresses in a thin semi-infinite rod due to some time dependent temperature applied to its free end

Bikas Ranjan Das
(Jadavpur University, Calcutta, India)

ABSTRACT : In this note is determined the distribution of temperature and stresses in a semi-infinite thin elastic rod when its free end is subjected to (a) an instantaneous rise in temperature and (b) a transient temperature distribution. The operational calculus has been used.

VOL. 9, NO. 4, 1961

Note on finite deformation of a hollow spherical shell of non-homogeneous material subjected to uniform normal tractions on the inner and outer surfaces

Miss Tara Ghosh

(Basanti Debi College, Calcutta, India)

ABSTRACT : Convergent analytic solutions are obtained for the governing equations in the problem of hollow spherical shell of non-homogeneous material subjected to uniform normal tractions on the inner and outer surfaces.

VOL. 9, NO. 4, 1961

A note on the longitudinal propagation of elastic disturbance in a thin inhomogenous elastic rod

S. P. Sur

(Vidyasagar College, Calcutta, India)

ABSTRACT : In this note, propagation of disturbance in an inhomogeneous rod of uniform density ρ has been considered. Here elastic parameters λ and μ are supposed to vary exponentially. A similar problem had been studied by Datta (1956), on the supposition that the elastic parameters have linear variation.

VOL. 9, NO. 4, 1961

VOLUME 10

1962

Theory of pair production without assuming Dirac's hole

K. C. Kar

(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : A new idea regarding the process of pair production has been suggested. The wavestatistical theory of

this process is developed, completely discarding Dirac's idea of 'hole'. The formula thus derived is in complete accord with that derived by Bethe-Heitler (Proc. Roy. Soc. Vol. 149, p. 83, 1934) and Racah (Nuov. Cim. Vol. 11, No. 7) using Dirac's theory and his artificial idea of 'hole'.

VOL. 10, NO. 1, 1962

Note on the vibration of thin transversly isotropic circular plate with a central hole

Subhas Dutta
(Bangabasi College, Calcutta, India)

ABSTRACT : The frequency equation of free vibration of a thin transversely isotropic circular plate has been obtained. The numerical solution of the equation is performed with free boundary at $r=1$ and $r=2$.

VOL. 10, NO. 1, 1962

Note on responses in a piezoelectric plate-transducer with a periodic step input

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : The electrical response corresponding to a mechanical force input having a time-dependance of the form $H(t) \cos \omega t$ (where $H(t)$ is the Heaviside's unit step function) and reciprocally, the mechanical response corresponding to a similar electrical voltage in a piezoelectric transducer have been worked out in the present note.

VOL. 10, NO. 1, 1962

Stresses due to a nucleus of thermo-elastic strain in an infinite elastic solid with a spherical elastic inclusion of a different material

S. C. Bose
(Chandernagore College, W. B., India)

ABSTRACT : Stresses in an infinite elastic solid due to a nucleus of thermo-elastic strain in presence of an elastic

spherical inclusion have been determined, the nucleus of thermo-elastic strain being situated inside or outside the spherical surface Numerical values of displacements and stresses on the surface of the inclusion for particular values of Poisson's ratio and the ratio of the moduli of rigidity of the two materials have been discussed.

VOL. 10, NOS. 2 & 3, 1962

A note on the small deformation in the interior of different earth models

S. K. Pan
(Calcutta Technical School, Calcutta, India)

ABSTRACT : B. A. Bolt proposed several earth models having different density distributions. In this paper the small strain theory has been applied to find out the deformation in the interior of the earth considered as a self-gravitating isotropic sphere of heterogeneous density distributions in Y_1, Y_2, Y_3, Y_4 of Bolt's earth models. Love also applied this theory to the same problem assuming the density to be uniform. The results in both cases have been compared.

VOL. 10, NOS. 2 & 3, 1962

Effect of a small circular hole on the stress distribution in a deep rectangular beam subject to flexure under shear

D. P. Gupta
(Birla Institute of Technology, Bihar, India)

ABSTRACT : The effect of a small circular hole on the stress distribution in a deep rectangular beam bent with shear is discussed, the centre of the hole being on the neutral axis. The solution is obtained in bipolar co-ordinates and the peripheral stress on the boundary of the hole for different positions of its centre on the neutral axis are found out.

VOL. 10, NOS. 2 & 3, 1962

Note on magneto-elastic stresses in a rotating cylinder acted on by a prescribed magnetic field

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : The present note seeks to work out stresses in a rotating cylinder acted upon by a prescribed magnetic field, by making use of the constitutive relations of a magnetostrictive material.

VOL. 10, NOS. 2 & 3, 1962

Note on the rotation of an aeolotropic elliptic cylinder about its axis

Dipti Dev
(Jadavpur University, Calcutta, India)

ABSTRACT : In this paper the problem of an aelotropic cylinder of elliptic cross-section steadily rotating about its axis has been solved.

VOL. 10, NO. 4, 1962

Note on the flow of a viscous fluid standing on a rigid base due to a single periodic pulse of tangential force on the surface

Panchanan Bhattacharyya
(Siliguri College, Darjeeling, W. B., India)

ABSTRACT : In this note, the motion of a viscous fluid has been investigated when a tangential pulse of a periodic force is applied on the surface. The motion has been assumed laminar.

VOL. 10, NO. 4, 1962

Note on the transient response of a linear visco-elastic plate in the form of an equilateral triangle

Bibhuti Bhusan Sen

(164, Jodhpur Park, Calcutta, India)

ABSTRACT : Using trilinear co-ordinates the problem of the dynamic response of a linear visco-elastic plate in the form of an equilateral triangle subjected to uniform load has been solved in this note.

VOL. 10, NO. 4, 1962

VOLUME 11

1963

Lorentz invariance and linearisation of the relativistic Hamiltonian

K. C. Kar

(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : Lorentz invariance of the electromagnetic equations and other fundamental equations have been proved by a new method. In giving the proof it has been shown incidentally that the charge density (ρ), the electric potential (ϕ) and the electron mass (m_o) are vectors in the direction of time. It is also shown that the magnetic intensity (H) and the acceleration are purely space vectors, whereas the electric intensity (E), the velocity (v) and the mechanical force on an electron (f) are both space and time vectors.

VOL. 11, NO. 1, 1963

Note on the deformation and stresses in an earth model with a rigid core

P. P. Chatterji and S. C. Bose
(Calcutta University & Chandernagore College, W. B., India)

ABSTRACT : The problem of the spherical earth with a rigid core has been discussed. The radial stress-component and displacement at different central distances have been numerically calculated.

VOL. 11, NO. 1, 1963

Note on problems of inclusion in an infinitely extended aeolotropic medium

Pran Ranjan Sengupta
(Ramakrishna Mission Vidyamandira, Howrah, W. B., India)

ABSTRACT : A spherical inclusion of a spherically aeolotropic medium is considered within another infinitely extended spherically aeolotropic medium. Secondly, a cylindrical inclusion of a transversely isotropic material is considered within another transversely isotropic infinitely extended medium.

VOL. 11, NO. 1, 1963

Relativistic variational principle for a charged hydromagnetic compressible fluid

S. R. Sharma and N. K. Sharma
(University of Rajasthan, Jaipur, India)

ABSTRACT : A relativistic energy principle for a charged, hydromagnetic, non-viscous and compressible fluid has been formulated in arbitrary co-ordinates. The variations of the variational parameters yield the equations of continuity, field equations and the equations of the fluid flow for the relativistic charged fluid.

VOL. 11, NO. 2, 1963

Stress distribution in an elastic sphere with a core of different elastic material rotating about a diameter

Miss Dipti Deb
(Jadavpur University, Calcutta, India)

ABSTRACT: A direct method has been employed in this paper to find stresses in a steadily rotating composite sphere with outer boundary free, the sphere having an outer shell of an isotropic material and an isotropic core of different type.

VOL. 11, NO. 2, 1963

Torsion of a large, thick non-homogeneous plate

R. N. Chatterjee
(Maulana Azad College, Calcutta, India)

ABSTRACT: The present paper deals with the problem of torsion of a non-homogeneous large thick plate rigidly fixed at one face subject to a torsional shearing force prescribed over a circular area on the other face, the modulus of rigidity varying as $\mu=\mu_0(1+kz)^n$, k being a constant and n being any rational number.

VOL. 11, NO. 2, 1963

Vectors in the theory of relativity

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT: It is shown that dx, dy, dz and dt forming the four dimensional manifold, are contravariant vectors. A_x, A_y, A_z and ϕ in electromagnetic theory are also contravariant four vectors. It is also shown that the charge density (ρ) and electronic mass (m) are contravariant vectors along time. The

gradient of a scalar along any of the four axes is a covariant vector in the direction of the gradient. However, the gradient along time, of a space vector is a space-time contra-covariant double vector. The electric intensity (E) and velocity (v) are shown to be double vectors of this type. But as the time gradient of a time vector is a time-scalar, acceleration $(f)=\frac{dv}{dt}$ is a space vector. However, force$=mf$ is a space-time double vector.

VOL. 11, NO. 3, 1963

Differential equations of the torsional vibration of tapered bars

W. R. Utz

(University of Missouri, Columbia, U. S. A.)

ABSTRACT : Differential equations arising from the torsional vibration of a bar or shaft of varying cross-section are solved in certain cases where it is assumed that the moment of inertia and torsional rigidity are proportional to the same function of the span-wise co-ordinate.

VOL. 11, NO. 3, 1963

Approximate solution of a one-dimensional coupled thermo-elastic problem

Ranjit Kumar Mahalanabis

(Jadavpur University, Calcutta, India)

ABSTRACT : This paper deals with the determination of the distribution of temperature and displacement in a thin semi-infinite elastic rod when its free end is subjected to periodic heating. The perturbation method has been used to give an approximate solution.

VOL. 11, NO. 3, 1963

Note on electrical and mechanical responses with ramp-type input signal in piezoelectric plate transducer

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : The electrical signal produced by a ramp function of mechanical force and the mechanical signal produced by a ramp-type voltage in a piezoelectric transducer have been obtained in the present note.

VOL. 11 NO. 4, 1963

Note on the torsional vibration of a thin beam of varying cross-section

Alok Kumar Sengupta
(Jadavpur University, Calcutta, India)

ABSTRACT : In the present note the problem of torsional vibration of a bar in which the moment of inertia and the torsional rigidity are proportional to $1+ks$ where s is the span-wise co-ordinate and k is a real number, has been considered.

VOL. 11, NO. 4, 1963

Generation of Love waves due to a point source in a layered medium

K. R. Nag
(Indian School of Mines, Dhanbad, India)

ABSTRACT : The problem of the generation of Love waves due to a point source in the upper layer overlying a half-space has been solved by a method involving Laplace and Hankel transforms and the displacement on the free surface obtained in closed form.

VOL 11, NO. 4, 1963

VOLUME 12

1964

আপেক্ষিক তাত্ত্বিক হ্যামিল্টন সূত্রের একঘাত করণ

কে. সি. কর
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : It is shown by the wavestatistical and relativistic methods that the electronic mass m_o is a vector in the direction of time. Accordingly the second order relativistic Hamiltonian is at once linearised by ordinary vector method. Dirac's method of linearising the Hamiltonian is wrong as he has vectorised scalars, namely, α_x, α_y, α_z and β by introducing convenient matrices. It is shown that if an alternative set of matrices is used then α_x, α_y, α_z and β retain their scalar property and obey the commutation law.

VOL. 12, NO. 1, 1964

On the study of stagnation points on a circular cylinder in contact with a straight wall in an ideal fluid

Sukumar Lahiri
(City College, Calcutta, India)

ABSTRACT : The problem of two-dimensional flow with constant shear past a circular cylinder in contact with a straight wall in an ideal fluid has been considered in this paper. The positions of the stagnation points for different values of a non-dimensional parameter have been determined.

VOL. 12, NO. 1, 1964

Note on the distribution of stresses in an isotropic solid cylinder clamped symmetrically and pushed longitudinally against friction

Gouri Das
(Bethune College, Calcutta, India)

ABSTRACT : In this paper the stresses in an isotropic circular cylinder clamped symmetrically at the ends of a diameter and pushed against friction along the axis have been discussed. This is an extension of the problem discussed by Filon, of a cylindrical roller gripped tightly between two parallel planes and forced through axially against friction.

VOL. 12, NO. 1, 1964

On the torsion of a circular cylinder of transversely isotropic material having a spherical inclusion of spherically isotropic material on its axis

S. C. Bose
(Chandernagore College, W. B., India)

ABSTRACT : The paper deals with the problem of torsion of a transversely isotropic circular cylinder having on its axis a spherical inclusion of spherically isotropic material. With the introduction of two functions-displacement function and stress function—the results have been obtained in closed forms.

VOL. 12, NO. 2, 1964

The nature of ether

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : It is shown that all the properties of ether, *e. g.*, its penetrability, transmissibility of transverse waves, exhibition of vacuum field etc. can be very satisfactorily explained if it is assumed that ether consists of a sea of electro-positrons in an annihilated state.

VOL. 12, NO. 2, 1964

Note on non-homogeneous rotating circular disk

Basanta Kumar Samanta
(Hooghly Mohsin College, Hooghly, W. B., India)

ABSTRACT : The problem is concerned with the stress distribution in a circular elastic solid disk which is rotating with a constant angular velocity about the axis through its centre and whose modulus of elasticity is an exponential function of the radial distance from the centre. The thickness of the disk and Poisson's ratio are taken as constants. Radial displacement is calculated and is found to be single-valued. A second formulation of the same problem is attempted by taking the radial displacement as the dependent variable in order to verify the results obtained from stress-function formulation. The hoop-stress is also determined for the disk with special reference to the case when Poisson's ratio is 0·3, as for structural steel.

VOL. 12, NO. 2, 1964

Note on the stress distribution due to a localised axial shear acting on the surface of a long circular cylinder of transversely isotropic material with a co-axial cylindrical rigid inclusion

Pran Ranjan Sengupta
(Ramakrishna Mission Vidyamandira, Howrah, W. B., India)

ABSTRACT : The problem of a localised axial shear on the external boundary of a long circular cylinder of transversely isotropic material with a co-axial cylindrical rigid inclusion has been considered in this paper.

VOL. 12, NO. 3, 1964

Note on stress waves in a rod of general type of linear substance

S. K. Sarkar
(Jadavpur University, Calcutta, India)

ABSTRACT : The object of this note is to find the stress in a semi-infinite rod of a general type of linear substance when an impulsive force is applied at one end.

VOL. 12, NO. 3, 1964

Note on the damped vibration of an anisotropic rectangular plate

S. P. Sur
(Vidyasagar College, Calcutta, India)

ABSTRACT : Using double infinite series, the problem of damped vibration of simply supported rectangular plate of an anisotropic material has been solved in this note.

VOL. 12, NO. 3, 1964

Vibration of a stiff string

S. K. Ghose and Sunil Kumar Banerjee
(Jadavpur University, Calcutta, India)

ABSTRACT : Dynamics of vibration of a stiff string of finite length excited by transverse impact by an inelastic load has been worked out following operational method. We consider in this paper the dynamics of such a string struck at the mid-point only. Two distinct cases have been worked out. (1) The tension of the string is large compared to stiffness. (2) Stiffness is large compared to the tension of the string. In case (1) series solution is obtained from which the solution for any epoch may be calculated easily, unlike the method of variation of Integration Constant, where solution for any particular epoch is obtained after a long laborious calculation. In case (2) a periodic solution is obtained.

VOL. 12, NO. 4, 1964

Thermo-elastic stresses in a rectangular sheet of transversely isotropic material due to a given temperature distribution on two opposite edges, other edges being insulated

Madhab Chandra Bagchi
(Surendranath College, Calcutta, India)

ABSTRACT : In this paper, stresses due to a given temperature distribution on two opposite edges of a rectangular plate of aelotropic materials other edges remaining insulated, have been considered.

VOL. 12, NO. 4, 1964

Note on finite radial deformation of a polarized annular disk of piezoelectric material

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : The present note intends to investigate the finite radial deformation of a polarized annular disk of piezoelectric material by applying Seth's well-known finite theory on deformation.

VOL. 12, NO. 4, 1964

VOLUME 13

1965

Theory of artificial transmutation of nuclei

K. C. Kar and A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In the present paper the authors have given a theory of the artificial disintegration of the nuclei assuming Yukawa type of extranuclear short range field as in their theory of the alpha disintegration (Ind. Jour. Theo. Phys. Vol. 8, p. 47, 1960). Physically both the problems are alike being that of the passage of the particle through the different regions of the nuclear field. The only difference is that in the natural disintegration the particle comes from inside the nucleus to the field-free region whereas in the present case the path is reversed. A more or less complete agreement has been obtained in both resonance and non-resonance type of disintegration.

VOL. 13, NO. 1, 1965

On the stresses due to a nucleus in the form of a centre of rotation in an elastic sphere embedded in an infinite elastic solid of a different material

S. N. Das

(Vidyasagar College for Women, Calcutta, India)

ABSTRACT : Stresses in an infinite elastic solid due to a nucleus in the form of a centre of rotation in presence of an elastic spherical inclusion have been determined, the nucleus being situated inside the spherical surface.

VOL. 13, NO. 1, 1965

Torsion of a non-homogeneous circular cylinder having a rigid spherical inclusion

R. N. Chatterjee

(Maulana Azad College, Calcutta, India)

ABSTRACT : Torsion of a non-homogeneous circular cylinder having a symmetrically located rigid spherical inclusion is discussed.

VOL. 13, NO. 1, 1965

On the stresses due to a nucleus of thermo-elastic strain at any point of an elliptic plate held fixed with a rigid non-conducting rim

S. K. Pan

(Calcutta Technical School, Calcutta, India)

ABSTRACT : The present paper deals with the determination of stresses due to a nucleus of thermo-elastic strain at any point of an elliptic plate held fixed with a rigid non-conducting rim.

VOL. 13, NO. 2, 1965

Dynamics of vibration of a cantilever, excited by a transversely impinging load (General Theory)

M. Ghosh and B. B. Banerjee
(City College, Calcutta, India)

ABSTRACT : In this paper the authors have developed a general theory of the dynamics of vibration of a cantilever, struck by a transversely impinging load at any point. The cantilever is assumed to be loaded so long as the load is in contact with it. The elastic hammer is taken to be inelastic load, backed by weightless spring. The cantilever is at rest before the impact begins. The theory is worked out by applying operational method to obtain expressions for (*i*) displacement of the struck point (*ii*) and pressure exerted by the hammer during impact. Ghosh-Roy's theory comes as a special case.

VOL. 13, NO. 2, 1965

Note on the buckling of non-homogeneous deep beam fixed at one end and free at the other, carrying a load at the free end

R. Bera
(Barasat Government College, Barasat, W. B., India)

ABSTRACT : The problem of buckling of deep beams of uniform cross-sections has been solved by Prescott. This paper deals with the problem of non-homogeneous beams fixed at one end and carrying a load at the other end which is free. It is attempted here to find the critical value of the load.

VOL. 13, NO. 2, 1965

তরঙ্গবেগের পরিমাপিকা রূপ

কে. সি. কর
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : Making use of the new conception of four dimensional vectors (*vide* Ind. Jour. Theo. Phys. Vol. 11, pp. 1,

75, 1963; ibid Vol. 12, p. 1, 1964) originating from the relativistic space-time continuum, it is shown mathematically that velocities of all kinds of waves found in nature, are *scalars*. Incidentally it is shown that magnetic pole strength is a vector in the direction of time. This may be easily realised when it is remembered that the magnetic pole is created by a spinning charge.

VOL. 13, NO. 3, 1965

Bending of uniformly compressed rectangular plate of variable thickness under lateral loads

Sukumar Basuli
(B. E. College, Howrah, W. B., India)

ABSTRACT : An attempt has been made in this paper to find the deflection of a uniformly compressed rectangular plate of linearly varying thickness under normal hydrostatic load. The method is an approximate one. Numerical calculations are given for a square plate.

VOL. 13, NO. 3, 1965

Various types of dislocation of an elastic cylinder whose cross section is bounded by two squares with rounded corners

A. Chakravarty
(Jadavpur University, Calcutta, India)

ABSTRACT : Dislocations of translation and rotation of a uniform cylinder with a section as mentioned in the title have been discussed in this paper. By choosing suitable complex potentials the boundaries of the cross section have been made stress free.

VOL. 13, NO. 3, 1965

Note on the transverse vibration of homogeneous rotating thin rods in the form of solids obtained by the revolution of a parabola of fourth order, a parabola of second order and a semi-cubical parabola about their axes

B. B. Kundu
(Bolpur College, Birbhum, W. B., India)

ABSTRACT : Applying methods after Southwell and Rayleigh-Ritz, the gravest modes of transverse vibration of some rotating thin rods are obtained. The configurations considered are solids formed by the revolution of a parabola of fourth order, second order and a semi-cubical parabola about the axis rotating about their thick ends.

VOL. 13, NO. 4, 1965

A note on the propagation of waves in a piezoelectric plate kept between two similar mechanical systems

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : The present note discusses the characteristics of a wave that propagates through a piezoelectric transducer.

VOL. 13, NO. 4, 1965

Note on the thermal stresses in an infinite slab resting on a rigid insulating plane and having a point heat-source outside the medium

Netai Chand Chattopadhyay
(Visva Bharati University, W. B., India)

ABSTRACT : The problem of determination of thermal stress in an elastic slab resting on a rigid insulating plane and having a point heat-source at a finite distance from the upper surface has been solved in this paper.

VOL. 13, NO 4, 1965

VOLUME 14

1966

সমষ্টি তরঙ্গবাদের মূলনীতি

(The fundamentals of Wavestatistics)

কে. সি. কর

(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The principles of wavestatistics developed by the writer have been discussed avoiding mathematics as far as possible. Why the particular differential equation discovered by Schrödinger gives the correct eigen-energy ? Why an exchange of positions or spins gives rise to a Heisenberg force ? What is the basis of the selection rule in line spectra ? Why, again, acoustical velocities c_l and c_t are used in Debye's theory of specific heat ? Explanations of all these questions will be found in the present article.

VOL. 14, NO. 1, 1966

A note on the problem of torsion of a magnetostrictive circular bar permeated by a longitudinal magnetic field

D. K. Sinha

(Jadavpur University, Calcutta, India)

ABSTRACT : The constitutive equations of a magnetostrictive material established by Lewis have been applied to investigate the torsion of a magnetostrictive circular bar.

VOL. 14, NO. 1, 1966

Note on the longitudinal propagation of elastic disturbance in a thin inhomogeneous elastic rod

S. L. Ghosh

(Bolpur College, Birbhum, W. B., India)

ABSTRACT : In this note the problem of longitudinal propagation of elastic disturbance in a rod exhibiting linear

variation of elastic parameters has been solved for two different end conditions *viz.*, (1) for a rectangular pulse of pressure P and T sec. duration and (2) a stress pulse of magnitude $S \sin \omega t$ applied at one end.

VOL. 14, NO. 1, 1966

Thermo-elastic stresses in a semi-infinite elastic strip of transversely isotropic material due to a given temperature distribution on one straight edge, two parallel edges bounding the strip being insulated

Madhab Chandra Bagchi
(Surendranath College, Calcutta, India)

ABSTRACT : In this paper, stresses due to a given temperature distribution on the straight edge of a semi-infinite elastic strip of a aelotropic material, with two infinitely long sides insulated, have been considered.

VOL. 14, NO. 2, 1966

Rotational motion of a non-Newtonian fluid through an annulus with porous walls

K. C. Bagchi
(Jadavpur University, Calcutta, India)

ABSTRACT : In this paper the motion of a non-Newtonian fluid between two co-axial circular cylinders is studied when the inner cylinder rotates about its axis with a transient angular velocity and the outer cylinder is at rest. It is observed that the effect of the cross-viscosity is absent in the velocity distribution and is shown up only in the pressure distribution. Finally, numerical results were obtained for a particular value of the cross-flow Reynolds number.

VOL. 14, NO. 2, 1966

Note on the flow of two incompressible immiscible viscous fluids due to the pulses of tangential force on the upper surface

P. Bhattacharyya
(North Bengal University, Darjeeling, W. B., India)

ABSTRACT: The problem of the laminar flow of two incompressible immiscible viscous fluids, one standing on the other, with prescribed tangential forces on the upper surface has been solved in this note.

VOL. 14, NO. 2, 1966

Note on the creep analysis for rotating solid disks of variable thickness

Dipti Dev
(Jadavpur University, Calcutta, India)

ABSTRACT: In this paper, creep analysis for stress-distribution in a solid disk of thickness varying exponentially with the distance from the centre and rotating uniformly about the normal axis has been discussed. Temperature has been assumed to be uniform. In this analysis Tresca criterion and its associated flow rule have been used.

VOL. 14, NO. 3, 1966

Note on stresses in a magnetostrictive composite rotating cylinder

R. R. Giri
(Jadavpur University, Calcutta, India)

ABSTRACT: The aim of the present note is to present the method of determining the magneto-elastic stresses developed in a composite rotating cylinder, subjected to a prescribed magnetic field.

VOL. 14 NO. 3, 1966

Note on the transverse vibration of a circular membrane of variable density

B. B. Kundu
(Bolepur College, Birbhum, W. B., India)

ABSTRACT: In this paper the problem of transverse vibration of a circular membrane with the density varying as $\exp(-\epsilon r)$ has been solved.

VOL. 14, NO. 3, 1966

Note on the longitudinal vibration of a paraboloidal bar

S. L. Ghosh
(Bolepur College, Birbhum, W. B., India)

ABSTRACT: In the present note a new problem namely that of longitudinal vibration of a homogeneous truncated paraboloidal bar has been discussed.

VOL. 14, NO. 4, 1966

Note on the flow of power-law fluid down an inclined plane between two parallel plates with uniform suction and injection

K. C. Bagchi
(Jadavpur University, Calcutta, India)

ABSTRACT: This paper deals with the flow of a power-law fluid down an inclined plane between two parallel plates when there is uniform suction at one plate and uniform injection at the other.

VOL. 14, NO. 4, 1966

Note on the study of shear flow past a cylinder whose cross-section is an inverse of an ellipse

A. K. Ghosh
(Visva-Bharati University, W. B., India)

ABSTRACT: The stream function and stagnation points on the cylinder whose cross-section is an inverse of an ellipse are discussed in this paper, the cylinder being placed in an ideal fluid with constant shear.

VOL. 14, NO. 4, 1966

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1967

Theory of Neutron induced disintegration of nuclei

A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : This paper presents a new theory of neutron-induced disintegration of nuclei assuming a short range extra-nuclear attractive potential of Yukawa type. A very simple formula has been deduced by solving the wave equation with the help of W. K. B. approximation method. The results obtained are found to follow the experimental observations very closely, both at resonant and non-resonant energies. The $\frac{1}{v}$ law has also been verified.

VOL. 15, NO. 1, 1967

On thermal stresses in a semi-infinite initially stressed solid due to prescribed temperature distribution on its surface

Shankar Prasad Bhattacharyya
(Vivekananda Centenary College, 24-Parganas, W. B., India)

ABSTRACT : In a semi-infinite isotropic solid, small deformations due to steady state temperature distribution have been superposed on a large deformation, the solid being initially stressed. The boundary surface is subjected to prescribed temperature distribution within a circular region, the rest of the surface being assumed to be free of any such distribution. The corresponding stresses generated within the body are determined and expressions for radial and cross-radial stresses have been obtained on the surface when the temperature distribution is either hemispherical or paraboloidal. Numerical results have also been deduced showing the variation of $[t_{\theta\theta}]_{z=o}$ for both the cases above for the particular material known as Mooney type material.

VOL. 15, NO. 1, 1967

Deflection of a square plate of variable thickness under a variable load and uniform tension in the middle plane of the plate

Barun Banerjee

(Jalpaiguri Engineering College, Jalpaiguri, W. B., India)

ABSTRACT : The present paper deals with the problem of a square plate of variable thickness under the simultaneous action of hydrostatic load and uniform tension in the middle plane of the plate.

VOL. 15, NO. 1, 1967

Note on the steady flow of viscous liquid between two cylinders whose cross-sections are confocal elliptic limacons, when the inner cylinder is towed along its axis

A. K. Ghosh

(Visva-Bharati University, W. B., India)

ABSTRACT : In this paper the velocity distribution in a viscous liquid between two cylinders has been discussed when the inner one is towed along its axis with an arbitrary uniform velocity. The cross-sections of the cylinders are so taken that they form confocal elliptic limacons. The shearing stress on the moving cylinder has also been obtained.

VOL. 15, NO 2, 1967

Transient temperature in a thick dielectric of spherical shape in an alternating current

R. Subramanian

(Indian Institute of Technology, Madras, India)

ABSTRACT : Analysis has been made about the transient temperature phenomena in a hollow thick-walled spherical dielectric between concentric spherical electrodes to which an alternating current is applied. It is assumed that the dielectric material is homogeneous, the electric and thermal fields are radially distributed, and the flow of heat is parallel to the direction of the electric field.

VOL. 15, NO. 2, 1967

Note on thermo-elastic stresses in a thin semi-infinite rod due to some time-dependent temperature applied to its free end

Netai Chand Chattopadhyay

(Visva-Bharati University, W. B., India)

ABSTRACT : In this note the temperature distribution and thermal stresses in a thin semi-infinite rod (a) when its free end is subjected to a constant temperature for a finite interval of time and kept at zero temperature afterwards and (b) when pulses of impulsive temperature are applied at the free end, have been determined.

VOL. 15, NO. 2, 1967

সমষ্টি তরঙ্গবাদের মূলনীতি

(আপেক্ষিক তত্ত্বের দিক হতে)

Fundamentals of Wave statistics (Relativistic)

K. C. Kar

(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In this paper a brief account is given of the different methods used in Wavestatistics for linearising the second order relativistic Hamiltonian. Of these the relativistic method shows that the linear Hamiltonian is an alternative form of Einstein's second order Hamiltonian. It has been shown by the writer in his book on Wavestatistics that on using this linear form Klein-Nishina formula and formulae of radiative scattering and pair production can very easily be deduced. Thus there is absolutely no necessity for Dirac matrices. It is shown that Dirac could get correct result through matrices only because of his mutually cancelling double mistakes. So the matrices should now be buried for ever.

VOL. 15, NO. 3 1967

Torsion of a large thick composite plate under shearing forces applied on two opposite faces with rigidity varying linearly with depth

P. R. Ghosh
(Vidyasagar Evening College, Calcutta, India)

ABSTRACT : Torsional stresses and displacements of a large thick composite plate having two types of elastic materials above and beneath the middle plane for which torsional shearing forces are prescribed on the two opposite faces, with rigidity varying linearly with depth, are obtained by expressing the applied shearing forces in terms of a Fourier-Bessel integral.

VOL. 15, NO. 3, 1967

A note on the magnetohydrodynamic two-layer viscous stratified flow down an inclined plane

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : In the present note, the flow of a two-layer viscous liquid permeated by an initial magnetic field has been investigated, making use of Laplace transforms.

VOL. 15, NO. 3, 1967

Stress distribution in an anisotropic circular thin disk rotating with variable angular velocity

Basanta Kumar Samanta
(Hooghly Mohsin College, Hooghly, W. B., India)

ABSTRACT : This paper is concerned with the problem of stress distribution in an anisotropic thin circular annular disk rotating non-uniformly. A transient shearing stress is applied on the outer edge. Some particular cases are also discussed.

VOL. 15, NO. 4, 1967

Note on the forced torsional vibration of a non-homogeneous cone with spherical caps twisted by periodic terminal couples

Sunita Mukherjee
(Visva-Bharati University, W. B., India)

ABSTRACT : In the present paper attempt has been made to solve the problem of forced torsional vibration of a non-homogeneous cone with spherical caps, where the density is supposed to be constant and the rigidity varies as the nth power of the distance from the vertex of the cone. Vibration is set up in the cone by applying equal and opposite periodic couples on the two spherical caps of the cone. The displacement produced is calculated from these conditions. The particular case, when the rigidity varies as the square of the distance from the vertex is treated separately.

VOL. 15, NO. 4, 1967

Note on the slow and steady rotation of a solid formed by the revolution of a cardioid about its axis in a viscous fluid

K. Sengupta
(Visva-Bharati University, W. B., India)

ABSTRACT : In this note the problem of slow and steady rotation of a solid made by the revolution of a cardioid about its axis in a viscous fluid has been studied.

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1968

On the theory of relativity

K. C. Kar and Mrs. C. Dutta
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In previous papers it has been shown by the wavestatistical and the relativistic method that the electron

mass 'm' is a vector in the direction of time. This vector property of 'm' has been utilised in the present paper to give a very simple proof of the formula $m=m_0\beta$ for the relativistic change of mass. It is also shown that there can be only one formula for the relativistic change of mass and that Einstein's conception of longitudinal and transverse mass is wrong. It has been pointed out that the introduction of Mincowski co-ordinate $x_4=ict$ has made the four dimensional space spherical leading to wrong conclusion that the electric intensities E_x, E_y and E_z are rotations in 14, 24 and 34 planes. This is not possible since we know that rotations in any plane cannot produce a vector in the same plane. Lastly, it has been proved that Lorentz's mechanical equation is invariant. The usual objection against invariance on the ground that the left-hand side of the equation is a vector while the right-hand side is a tensor is not tenable since in our theory the electron mass is a vector and so the left-hand side is also a tensor.

VOL. 16, NO. 1, 1968

A note on finite deformations of a composite cylinder of barium titanate

Hari Sankar Chakrabarty
(Kanyapur Polytechnic, Burdwan, W. B., India)

ABSTRACT: The present note makes use of the electromagnetic equations of Maxwell, equations of elasticity to investigate the deformation of a cylinder made of barium titanate.

VOL. 16, NO. 1, 1968

A note on responses of a piezoelectric plate-transducer owing to prescribed inputs

N. C. Das
(Jadavpur University, Calcutta, India)

ABSTRACT: The problem stated in the title has been solved by using the principles of transform calculus.

VOL. 16, NO. 1, 1968

On the bending of an annular plate of variable rigidity under the combined action of uniform compression and uniform load

P. K. Ghosh
(Visva-Bharati University, Birbhum, W. B., India)

ABSTRACT : In this paper, the deflexion of a thin annular plate of variable rigidity possessing rotational symmetry under the simultaneous action of uniform compression in the middle plane of the plate and uniform lateral loading has been solved when its flexural rigidity varies directly as the square of the distance from the centre and numerical solution has been obtained for a particular case.

VOL. 16, NO. 1, 1968

Radial deformations of two concentric layers of ceramic intervened by a concentric layer of piezoelectric quartz

Sachindra Kumar Bakshi
(Ramakrishna Mission Vidyamandira, Howrah, W. B., India)

ABSTRACT : The paper deals with the problem of determining the radial deformations of two concentric layers of ceramic intervened by a concentric layer of piezoelectric quartz subjected to the joint effect of heating and to a distribution of charge along the inner boundary of the inner ceramic layer.

VOL. 16, NO. 2, 1968

A note on the slow and steady flow of viscous fluid within two rotating confocal prolate spheroid

K. Sengupta
(Visva-Bharati University, Birbhum, W. B., India)

ABSTRACT : Slow and steady flow of incompressible viscous fluid contained within two confocal prolate spheroids due to the rotation of the boundaries is discussed in the note. An expression for moment on the outer boundary is also calculated.

VOL. 16, NO. 2, 1968

Locus of the image of a point-object due to liquid surfaces swept over by waves

Arun Kumar Gupta
(Jadavpur University, Calcutta, India)

ABSTRACT : Considering the angular velocity of a plane mirror about an axis lying in the plane of the mirror, the instantaneous position of the image of a point-object has been found out. The effect of superimposition of a sinusoidal angular velocity combined with a linear velocity in a direction perpendicular to the plane of the mirror has been investigated. Taking into account an element perpendicular to the direction of propagation of the wave, its sinusoidal angular motion and sinusoidal linear motion have been calculated from the wave equation for the surface waves. The characteristics of these two motions have been applied to determine the instantaneous position of the image of the point-object kept above the undulating surface. The characteristics of the locus for smaller and larger wavelengths have also been investigated.

VOL. 16, NO. 2, 1968

On finite deformations of a long rotating cylinder of piezoelectric material

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : The present note aims at solving the problem stated in the title using Seth's theory of deformation and usual equations of elasticity, electro-magnetic equations of Maxwell.

VOL. 16, NO. 2, 1968

On an approximate form of H-functions for isotropic scattering (non-conservative)

Sitansubimal Karanjai
(University of North Bengal, Darjeeling, W. B., India)

ABSTRACT : In this short note is given an useful first approximation of $H(\tilde{\omega}_o, \mu)$ for small values of $\tilde{\omega}_o$ not exceeding. 0·4 with the help of this first approximation we have calculated contour of a hypothetical faint absorption line.

VOL. 16, NO. 3, 1968

A note on the deformation of an electrostrictive prismatical bar

P. Roy

(Victoria College, Cooch Behar, W. B., India)

ABSTRACT : The present note considers the static deformation of an electrostrictive prismatical bar under the influence of a constant electric field in the longtiudinal direction.

VOL. 16, NO 3, 1968

Torsion of a hollow circular composite cylinder of cylindrically aeolotropic material under tangential tractions acted on the inner surface, outer surface being fixed

P. R. Ghosh

(Vidyasagar Evening College, Calcutta, India)

ABSTRACT : Torsional stresses and displacements in a hollow circular composite cylinder having two types of cylindrically aeolotropic elastic materials with the outer surface of the cylinder being fixed and the inner surface being acted on by tangential tractions, are obtained in this paper. The case when the tractions are distributed uniformly over the portions of the inner boundary is also considered. Lastly the similar problem for the isotropic case is discussed.

VOL. 16, NO. 3, 1968

Note on the forced torsional vibration of an aeolotropic cylinder having periodic shearing forces along a ring on the curved surface

D. Chatterjee

(Prabhu Jagatbandhu College, Howrah, W. B., India)

ABSTRACT : This note deals with the problem of forced vibration of a right circular cylinder of transversely isotropic material as a result of a shearing stress applied along the circumference of a circle on the surface of the cylinder. Solutions have been obtained when both the ends of the cylinder are fixed and when one end is fixed, the other end being free.

VOL. 16, NO. 3, 1968

On the propagation of gravitational fields through non-empty spaces

R. A. Singh
(Gorakhpur University, Gorakhpur, India)

ABSTRACT : A co-variant technique for the propagation of gravitational fields for non-empty spaces has been given. This technique is entirely based on the algebraic and differential properties of the Petrov space-time-matter tensor P_{abcd} and the geometry of the null congruences. It has been shown that the rays of Petrov type III and type II non-empty gravitational fields are not shear-free null geodesics as they are in the case of empty spaces. Optical properties of the rays depend entirely on the properties of the medium through which the rays are propagating.

VOL. 16, NO. 4, 1968

Stresses due to a nucleus of thermo-elastic strain in a composite ring

S. C. Bose
(University College of Science, Calcutta, India)

ABSTRACT : In this paper solution has been obtained of the problem of action of a nucleus of thermo-elastic strain in a composite circular ring. The solution is obtained in the form of series under the assumption of plane strain. Several particular cases have been discussed.

VOL. 16, NO. 4 1968

On the deformation and stresses in an earth model with a rigid core

S. R. Maiti
(Surendranath College, Calcutta, India)

ABSTRACT : In this paper the small strain theory has been applied to find out the deformation and stresses in the interior

of the earth considered as a self-gravitating isotropic sphere of heterogeneous density distribution obeying Roche's law, assuming one of the elastic parameters *viz.* λ to vary parabolically. In the light of the same theory the case of uniform density distribution in the interior of the earth has also been considered.

VOL. 16, NO. 4, 1968

Stresses due to a small elliptic hole on the neutral axis of a deep rectangular beam subject to flexure under shear

C. B. Mishra
(Birla Institute of Technology, Bihar, India)

ABSTRACT : In this paper the stresses due to a small elliptic hole, the major axis of which is inclined at an angle θ to the neutral axis of a beam under shear have been investigated. From the results obtained stress concentrations around circular and elliptic holes of simpler particular cases of interest are studied.

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1969

Notes on the theory of relativity

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In the paper some confusions still existing in the theory of relativity have been pointed out. It is shown that if the general definition of covariant vector previously given by the writer be adopted then the vector characters of any

quantity we meet with, in physical problems can be easily determined. As a result the mass and electrostatic potential which were known to be scaler are found to be really vectors in the direction of time. Consequently it is easy to group together four vectors and linearise the quadratic Hamiltonian of Einstein. Thus it is unnecessary to use Dirac's linear Hamiltonian. It is also shown that there may be bending of light in a strong gravitational field. But in the absence of this field there can be no bending. Lastly it is pointed out that Einstein world is restricted and one can imagine worlds in which Einstein theory is not at all applicable.

VOL. 17, NO. 1, 1969

A note on shallow-liquid magneto-hydrodynamic disturbances

M. C. Baral
(Tripura Engineering College, Tripura, India)

ABSTRACT : The present note seeks to determine the magneto-hydrodynamic disturbances in a shallow-liquid due to a transient exciting force.

VOL. 17, NO. 1 1969

Note on the stresses in a non-homogeneous rectangular plate under normal load at one edge

Karuna Kanta Ray
(City College, Calcutta, India)

ABSTRACT : In this note stresses in a non-homogeneous rectangular plate due to a normal load on one of its edge have been obtained. The opposite edge is supposed to be fixed while the side edges are in contact with smooth rigid panels. For non-homogeneity, exponential variation of Young's modulus with the distance from the loaded end is considered here.

VOL. 17, NO. 1, 1969

Radial deformation of a composite spherical shell of electrostrictive material under internal and external pressure

N. C. Das

(Jadavpur University, Calcutta, India)

ABSTRACT : The present note is an attempt to determine the deformation of a composite spherical shell of electrostrictive material when it is subjected to prescribed constant pressures on its inner and outer surfaces.

VOL. 17, NO. 1 1969

Unsteady spherical flow of ideal gases through porous medium of pressure—dependent permeability

S. K. Jain

(Shri Govindram Seksaria Technological Institute, Indore, M. P., India)

ABSTRACT : Assuming a sudden generation of large amount of the pressure at the centre of the porous medium, we obtain the pressure distribution throughout the medium. An expression for the shock wave front is obtained. It is observed that as the time progresses the disturbance is propagated throughout the medium, decreasing in intensity as it propagates.

VOL. 17. NO. 2, 1969

Transverse vibration of an anisotropic rectangular plate with a concentrated mass placed on it

B. B. Kundu

(Bolpur College, Birbhum, W. B., India)

ABSTRACT : The problem of transverse vibration of a non-isotropic rectangular plate of moderate thickness vibrating harmonically carrying a mass with it has been considered. The angular frequencies including the gravest one, have been obtained by using Dirac's delta function and applying Fourier finite sine transform. A numerical result is obtained for the angular frequency, corresponding to the gravest mode, of vibration of a square plate of moderate thickness.

VOL. 17, NO. 2 1969

Effect of couple-stress in a semi-infinite plate having a simply type of crack on the straight boundary

Rathindra Nath Das
(Krishnanagar Govt. College, Nadia, W. B., India)

ABSTRACT : The present paper is concerned with the effect of couple-stress in a semi-infinite elastic media due to the displacement prescribed on a certain portion of the straight boundary.

VOL. 17, NO. 2, 1969

Displacements in a rotating shaft of non-homogeneous piezoelectric material

B. Chaudhuri
(Shyampur College, W. B., India)

ABSTRACT : This note involves calculations of stresses, displacements and electric field in a rotating thick walled hollow cylindrical shaft of non-homogeneous piezoelectric material.

VOL. 17, NO. 2, 1969

Thermo-elastic stresses in an infinite plate having a pair of insulated unequal circular holes

S. R. Maiti
(Surendranath College, Calcutta, India)

ABSTRACT : The present paper deals with the determination of thermal stresses in an infinite elastic plate having a pair of insulated unequal circular holes by using bipolar co-ordintes—the stresses being caused by the disturbance of uniform heat flow in the body by the pair of holes. Numerical values for the non-vanishing stress component on some points of the circular boundaries have been calculated.

VOL. 17, NO. 3, 1969

Torsion of a prismatical bar with Cauchy's initial axial tension

D. K. Wagh

(Shri Govindram Seksaria Technological Institute, Indore, M. P., India)

ABSTRACT : Cauchy theory of initial stress has been applied to the problem of torsion of a prismatical bar which is under initial axial tension. It is found that the existence of initial axial tension alters the classical shear stress distribution and increases the torsional stiffness of the bar. The increase in torsional stiffness is found to be proportional to the polar moment of inertia of the cross-section with respect to its centre of gravity.

VOL. 17, NO. 3, 1969

On heat transfer in mhd Couette flow of a rarefied gas

V. M. Soundalegekar

(Indian Institute of Technology, Bombay, India)

ABSTRACT : An analysis of heat transfer in a mhd Couette flow of a viscous, incompressible, electrically conducting rarefied gas is presented here, under first-order velocity-slip and temperature-jump boundary conditions. Expressions for the Nusselt numbers N_{u1} (at the upper plate) and N_{u2} (at the stationary plate) are derived in closed form. It is observed that N_{u1} increases with increasing M (Hartmann number), K (an electric field parameter) and λ (slip-parameter), but decreases with Γ (temperature jump coefficient). N_{u2} decreases with M, λ and Γ, but increases with K.

VOL. 17, NO. 3, 1969

A note on the dynamics of vibration of a cantilever placed in a magnetic field

Miss H. Sinha

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ABSTRACT : In this paper, when the magnetic field is taken into consideration, the dynamics of the transverse vibration of a cantilever has been discussed.

VOL. 17, NO. 3, 1969

Concentration of stresses due to cracks and flaws in a solid under longitudinal shear

Gouri Demunshi
(Bethune College, Calcutta, India)

ABSTRACT : In this paper the problem of determination of stresses around small cavities in an infinite solid under longitudinal shear has been reduced to the ordinary potential problem. Solutions in closed forms are then obtained for three simple cases by using curvilinear co-ordinates.

VOL. 17, NO. 4, 1969

Note on the steady flow of elastico-viscous liquid between two circular cylinders when the inner cylinder is towed along its axis

Arun Kumar Ghosh
(Jadavpur University, Calcutta, India)

ABSTRACT : This paper considers the motion of an elastico-viscous liquid between two co-axial cylinders when the inner cylinder is started moving along its axis with an arbitrary constant velocity. The subsequent velocity distributions of the liquid and the skin friction on the moving cylinder have been obtained.

VOL. 17, NO. 4, 1969

Thermal stresses in an infinite elastic plate having a pair of insulated circular holes

P. R. Ghosh
(Vidyasagar Evening College, Calcutta, India)

ABSTRACT : Thermal stresses in an infinite elastic plate having a pair of insulated circular holes are obtained by using bipolar co-ordinates. In the case considered, the stresses are caused by the disturbance of uniform heat flow in the body by the pair of holes. Numerical values of the non-vanishing stress on the circular boundaries have been found.

VOL. 17, NO. 4, 1969

Note on higher derivatives for problems involving a singular solution

O. E. Widera and R. Amon
(University of Illinois, Chicago, U. S. A.)

ABSTRACT: In this paper higher derivatives in polar co-ordinates of bi-harmonic functions are given in terms of their complex potentials.

VOL. 17, NO. 4, 1969

VOLUME 18

1970

Relativity in the acoustical world

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT: In the theory of relativity propounded by Einstein, the velocity of light has been taken as the fundamental velocity in building the structure of the space-time continuum. It is assumed that the light velocity c is invariant in S and S' systems. These assumptions evidently lead to the conclusion that Einstein's world is purely optical, being meaningless to a person who is born blind.

It is shown in this paper that just like the optical world one can build acoustical worlds for different media on taking the corresponding sound velocity c_s as the fundamental velocity. In these acoustical worlds the velocities c_s's are shown to be invariant in S and S' systems. Hence Lorentz relations with c_s in place of c, are easily deduced. With the help of these relations for acoustical relativity, formulae for longitudinal and transverse Döppler effects are derived. Formulae for relativistic changes of mass, momentum and energy are also deduced. The relativistic Hamiltonian in the quadratic form has been linearised by Kar method. It is pointed out that the linear Hamiltonian gives partial canonical equations which are same for all the four components.

VOL. 18, NO. 1, 1970

Torsional vibration of non-homogeneous cylindrically aeolotropic cylindrical shell

P. R. Ghosh
(Vidyasagar Evening College, Calcutta, India)

ABSTRACT : In this paper, the problem of torsional vibration of non-homogeneous cylindrically aeolotropic cylindrical shell is discussed. The elastic constants c_{ij} are assumed to be of the form $c_{ij} = \mu_{ij}{}^{rn}$, μ_{ij} and n being constants and the density ρ of the material is assumed to be of the form $\rho = \rho_0 r^n$, ρ_0 being constant. The result for the non-homogeneous isotropic case and that for the homogeneous cylindrically aeolotropic case are obtained from this paper as special cases. Numerical values of frequencies are given.

VOL. 18, NO. 1, 1970

Radial deformation of an annular piezoelectric disk with a concentric inhomogeneous annular rigid inclusion and a symmetrical temperature distribution

B. Yadav
(Gorakhpur University, Gorakhpur, India)

ABSTRACT: The present note seeks to investigate the radial deformation of an annular inhomogeneous piezoelectric disk subjected to a symmetrical temperature distribution.

VOL. 18, NO. 1, 1970

Twisting of a transversely isotropic circular cylinder embeded in a resisting medium

N. Bhanja
(Ramakrishna Mission Residential College,
24-Parganas, W. B., India)

ABSTRACT : In this problem the stresses and displacements in the series form have been obtained throughout a circular cylinder made of transversely isotropic homogeneous elastic material embeded in a resisting medium, one end of it being kept fixed and the other end subjected to a shearing force.

VOL. 18, NO. 1, 1970

Effect of Cosserats' couple-stress on the stress distribution in a semi-infinite medium with varying modulus of elasticity

P. K. De
(Calcutta Technical School, Calcutta, India)

ABSTRACT : The theory of Cosserats' couple stress is briefly described in cartesian co-ordinates and the effect of couple-stresses on the stress distribution in a semi-infinite medium with varying modulus of elasticity has been considered.

VOL. 18, NO. 2, 1970

Flow of a conducting viscoelastic fluid between two parallel plates under a transverse magnetic field

P. N. Tandon
(Harcourt Butler Technological Institute, Kanpur, India)

ABSTRACT : In this paper, non-steady flow of a conducting viscoelastic fluid in between two parallel plates has been studied. It has been assumed that no applied or polarization voltages exist *i. e.* no energy is being added or extracted by electrical means. It has been found that unlike ordinary viscous fluid, in viscoelastic fluid periodic oscilliations are set up, the amplitudes of which decay with time. Various limiting cases have also been discussed.

VOL. 18, NO. 2, 1970

Note on the forced torsional vibrations of non-homogeneous spherical and cylindrical shells

D. M. Mishra
(Regional Engineering College, Orissa, India)

ABSTRACT : This paper deals with the torsional vibration of non-homogeneous thick spherical and cylindrical shells under periodic shearing forces on the outer boundary. Different cases are considered according as the shearing forces are prescribed over the part or whole of the outer surface. Here laws of

non-homogeneity are $\rho=\rho_0 r^n$ and $\mu=\mu_0 r^n$, where ρ and μ are density and modulus of rigidity of the material. In each case torsional displacement is determined in the form of an infinite series.

VOL. 18, NO. 2, 1970

On heat transfer in MHD channel flow between conducting walls in slip-flow regime

V. M. Soundalgekar
(Indian Institute of Technology, Bombay, India)

ABSTRACT : Heat transfer aspect of the laminar, fully developed flow of an electrically conducting incompressible rarefied gas in a conducting parallel-plate channel and under a constant external magnetic field is studied. Under slip-flow and temperature jump boundary conditions, closed form solutions for temperature, mean mixed temperature and Nusselt number are derived. It is observed that the temperature T, mean mixed temperature T_b increase with increasing h_1 (rarefaction parameter) but N_u decreases. T, T_b increase with increasing T but N_u decreases. T, T_b and N_u decrease as $\phi_1+\phi_2$ increases.

VOL. 18, NO. 2, 1970

Torsional vibration of a composite orthotropic cylinder in a magnetic field

Bireswar Ray Chaudhuri
(Howrah Zilla School, Howrah, W. B., India)

ABSTRACT : The problem of torsional vibration of a composite elastic cylinder in the presence of a magnetic field is considered. Both the inner and the outer materials are supposed to be orthotropic. The vibrations of a composite solid cylinder and a composite cylindrical shell have been investigated.

VOL. 18, NO. 3, 1970

Simple cases of torsional vibration of non-homogeneous circular cylinder

Sunita Mukherjee

(Surul, P. O. Sreeniketan, Birbhum, W. B., India)

ABSTRACT : Torsional vibration of a non-homogeneous circular cylinder has been discussed here. Non-homogeneity of rigidity and density in terms of cosine function (whose argument is αz, α being an arbitrary constant, z being the axial distance) has been considered. Non-homogeneity of both the constants is considered in case I, while we assume that only the rigidity varies in the case II. Numerical calculations have been carried out for finding out the frequencies of vibration in suitable cases.

VOL. 18, NO. 3, 1970

Effect of couple-stresses in a non-homogeneous semi-infinite elastic medium

G. R. Tiwari

(Shri G. S. Technological Institute, Indore, M. P., India)

ABSTRACT : Effects of couple-stresses on the stress distribution in a semi-infinite elastic medium with varying modulus of elasticity have been investigated. The method of successive approximations has been used as mathematical tool. Numerical results are also given.

VOL. 18, NO. 3, 1970

Note on the thermoelastic stress in a thin rod of finite length due to some constant temperature applied to its free end, the other end being fixed and insulated

Snehanshu Kumar Roy Choudhuri

(Jadavpur University, Calcutta, India)

ABSTRACT : This note is concerned with the determination of temperature and stress in a thin rod of finite length when its free end is subjected to a constant temperature and the other end which is insulated is kept fixed.

VOL. 18, NO. 3, 1970

Inclusions in elastic-plastic solids of workhardening material extended to a finite limit

P. R. Sengupta
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and

H. N. Kundu
(Jaipuria College, Calcutta, India)

ABSTRACT : This paper deals with the spherical and circular inclusions in an elastic-plastic solid of work-hardening material of finite extent in the case of small elastic-plastic deformations. The case of ideal plastic solid has been deduced from this general result.

VOL. 18, NO. 4, 1970

Mechanical response in a free piezo-electric plate subjected to a constant flow of heat

D. R. Ray
(Hindu College, 24-Parganas, W. B., India)

ABSTRACT : This paper investigates the mechanical response in a free piezo-electric plate with a time decaying polarisation gradient and with a constant flow of heat.

VOL. 18, NO. 4, 1970

On heat transfer in mhd Couette flow between conducting walls in slip-flow regime

V. M. Soundalgekar
(Indian Institute of Technology, Bombay, India)

ABSTRACT : An analysis of heat transfer in fully developed mhd Couette flow of an electrically conducting, viscous incompressible rarefied gas flowing between electrically conducting plates is presented. Closed form solutions are obtained. Temperature profiles are shown graphically while the numerical values of the Nusselt number Nu, are entered in Table. Effects of different parameters on Nu are discussed in conclusion.

VOL. 18, NO. 4, 1970

Note on problems of inclusion of inhomogeneous aeolotropic elastic solid in an infinitely extended aeolotropic material

Amarnath Basumullick
(New Aalipore College, Calcutta, India)

ABSTRACT : In this paper the problems of inclusion of inhomogeneous aeolotropic elastic solid in an infinitely extended aeolotropic material have been solved. Two different types of inhomogeneity of the inclusion have been discussed and the results are obtained in a closed form.

VOL. 18, NO. 4, 1970

Geodesie in the theory of relativity

K. C. Kar and A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

VOL. 18, NO. 4, 1970

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1971

On the origin of the gravitational field

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : Forces are found to act in nature in apparently five different forms. They are (1) electrical (2) magnetic (3) mechanical (4) statistical or wave statistical and (5) gravitational. It is shown that of these the magnetic, mechanical and gravitational forces are of electrical origin. This interpretation has explained that the gravitational field is always attractive and is never repulsive. Also the bending of light in gravitational field is explained. Einstein's hypothesis of curvature in space-time explains bending but does not explain

why the gravitational force is never repulsive. Because, according to the present theory gravitational field is electrical in nature there is no necessity of developing any unitary field theory.

The statistical or the wavestatistical force is a new kind of force. It is shown that these forces are consistent with the Indian philosophical idea that every inanimate object possesses three virtues, namely, Tamoguna (তমোগুণ), Rajoguna (রজোগুণ) and Sathwaguna (সত্বগুণ).

VOL. 19, NO. 1, 1971

Stresses in a non-homogeneous infinite elastic slab under normal load distributed on the free surface

Karuna Kanta Ray
(City College, Calcutta, India)

ABSTRACT : In this paper the problem of a non-homogeneous elastic slab, resting on a rigid welded foundation having normal pressure on the free boundary, is solved by the method of Hankel transforms. The modulus of rigidity of the medium varies exponentially with the distance from the fixed boundary.

VOL. 19, NO. 1, 1971

Flow of a viscoelastic fluid in a hydrodynamic wedge

P. N. Tandon
(Harcourt Butler Technological Institute, Kanpur, India)

ABSTRACT : The flow of a viscoelastic fluid represented as B'' by walters has been discussed in this paper. The problem has been solved using the basic assumptions of lubrication and taking the average value of the viscoelastic terms in the direction of the film thickness. The load-carrying capacity and the pressure distribution have been determined and it has been found that the load capacity decreases as the viscoelastic parameter increases.

VOL. 19, NO. 1, 1971

Torsion of a non-homogeneous plate

Smarajit Roy
(Visva-Bharati University, Birbhum, W. B., India)

ABSTRACT : The present paper deals with the problem of torsion of a non-homogeneous large thick plate rigidly fixed at one face subject to shearing force prescribed on the other face, the modulus of rigidity varying as

$$\mu=\mu_0 e^{-\mu_1' z^2},$$

μ_0 and μ_1 being constants.

VOL. 19, NO. 1, 1971

আপেক্ষিক তত্ত্ব সম্বন্ধে কয়েকটি মন্তব্য

(Notes on the theory of relativity)

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

VOL. 19, NO. 1, 1971

Theory of spontaneous alpha disintegration

A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In the present paper the problem of spontaneous alpha emission from natural radioactive nuclei has been discussed with the idea of (*i*) existence of a short range extranuclear force between the emitted alpha particle and the daughter nucleus, (*ii*) phase fluid damping inside the nucleus causing alpha emission. W. K. B. approximation has been adopted for solving the wave equations. The disintegration constant determined with the above assumptions is very simply related with charge and mass of the daughter nucleus and energy of the alpha particle. Satisfactory agreement has been obtained with the experimental results.

VOL. 19, NO. 2, 1971

Note on thermal stresses in an elastic sphere due to constant temperature and uniform normal pressure prescribed over a band along the equator

Jagadis Chandra Misra
(Uluberia College, Howrah, W. B., India)

ABSTRACT : The problem of a solid sphere of isotropic material with prescribed temperature and normal pressure on a portion of its curved surface along the equator is investigated in this paper.

VOL. 19, NO. 2, 1971

Mechanical response in a piezoelectric transducer subjected to a current flowing in a semi-conducting boundary layer characterised by a polarization gradient

Alok Chakrabarty
(S. A. Jaipuria College, Calcutta, India)

ABSTRACT : This note aims at determining the mechanical response in a rigidly backed piezo-electric transducer subjected to a current flowing in a semi-conducting boundary layer. The transducer is characterised by a polarization gradient and is subjected to a constant voltage input. The use of Laplace transform makes the solution of the problem a tractable one.

VOL. 19, NO. 2, 1971

A mixed boundary value problem of elasticity for an infinite plate with a crack along a circular arc

Aqueel Ahmed
(Aligarh Muslim University, Aligarh, U. P., India)

ABSTRACT : The solution of the problem of an infinite plate with a crack along a circular arc has been obtained when the outer segment of the crack is subject to constant displacement and the inner segment is stress free. The problem has been reduced to Hilbert equations of holomorphic functions of complex variable and is solved in a closed form.

VOL. 19, NO. 2, 1971

O_2 Emission in the night airglow of the martian atmosphere

D. C. Agarwal
(Allahabad University, Allahabad, India)

ABSTRACT : In this paper, the emission of the atmospheric and Herzberg systems of O_2 in the Martian atmosphere are calculated. It is concluded that the heights of emitting layer of O_2 Herzberg and atomspheric systems are about 60-80 km.

VOL. 19, NO. 3, 1971

Stability of small twist on homogeneous cylinders

M. Parameswara Rao and B. Kesava Rao
(P. G. Centre, Warangal, A. P., India)

ABSTRACT : Taking the criteria for stability of finite deformation of an elastic solid from Pearson, Hill and Green and Adkins, the condition for stability is obtained in the case of "Small twist superposed on a homogeneous prestressed cylinder subjected to a finite deformation" using the linear stress-strain relation.

VOL. 19, NO. 3, 1971

Thermal stresses in a long circular cylinder horizontally placed with its curved surface exposed to Sun's rays

J. C. Misra
(Uluberia College, Howrah, W. B., India)

ABSTRACT : This paper deals with the problem of a long circular cylinder of isotropic material heated by radiation from a distant source, the cylinder being placed in the horizontal position.

VOL. 19. NO. 3, 1971

Effect of radial magnetic field on the heat transfer in a viscoelastic fluid in an annulus

P. N. Tandon
(H. B. Technological Institute, Kanpur, India)
and
M. D. Raisinghania
(V. S. S. D. College, Kanpur, India)

ABSTRACT: Approximate analytical solutions have been obtained for the heat transfer in a conducting viscoelastic fluid in an annulus of two concentric cylinders in presence of radial magnetic field. The fluid is being injected at inner cylinder and withdrawn at the outer cylinder uniformly and a constant pressure gradient is acting in the direction of the common axis of the cylinders. It has been found that in the absence of suction or injection the fluid behaves as ordinary viscous fluid and the increasing values of Eckert and Hartmann numbers have opposite effects on the temperature profile. The Nusselt number is also found to depend on the parameters involved. Some of these results have been shown in the tables.

VOL. 19, NO. 3, 1971

On a new method of measurement of self-inductance of a transformer coil by a moving coil galvanometer

Arun Kumar Gupta and S. D. Chatterjee
(Central Scientific Instruments Organisation, Calcutta, India)

ABSTRACT: Utilising only predetermined galvanometer constants, a method has been developed for measuring the self-inductance of a transformer coil without the help of conventional impedance bridge circuits.

VOL. 19, NO. 4, 1971

A note on mechanical response in an inverted mesa piezoelectric structure

D. K. Sinha
(Jadavpur University, Calcutta, India)

ABSTRACT : The paper makes use of the governing equations of an inverted mesa structure together with Laplace transform to find at the mechanical response emitted owing to an appropriate electrical input.

VOL. 19, NO. 4, 1971

Torsional oscillation of an aeolotropic elastic cylinder placed in a magnetic field

Gouri Prasad Mukherjee
(Hindu School, Calcutta, India)

ABSTRACT : Here the problem is to determine the torsional oscillation of a cylinder made of aeolotropic elastic material placed in a magnetic field.

VOL. 19, NO. 4, 1971

Hydromagnetic disturbance produced by a momentary current in a semi-infinite fluid of finite electrical conductivity

K. P. Das
(Jalpaiguri Govt. Engineering College, Jalpaiguri, W. B., India)

ABSTRACT : The perturbations in velocity and magnetic field due to a momentary and infinitesimal current element in a semi-infinite fluid are derived. The fluid is supposed to be incompressible and finitely conducting. A uniform magnetic field is present perpendicular to its boundary. The results are given in integral form.

VOL. 19, NO. 4, 1971

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1972

Perihelion motion of planets and the new theory of gravitation

K. C. Kar and A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In the present paper Einstein's formula for the advance of perihelion of planets has been deduced following his simple theory of relativity, without taking any help of the complicated so called generalised theory of relativity. In the new theory of gravitation (Ind. Jour. Theo. Phys. Vol. 19, p. 1, 1971) it was shown that the gravitational force is always attractive and that the law of attraction is same as in classical mechanics. On using this law and introducing relativistic correction in the law of area, Einstein's formula for perihelion advance has been deduced in a very simple manner. It is thus shown that Einstein's general theory is not necessary for solving this problem.

VOL. 20, NO. 1, 1972

A note on pure bending of an electrostrictive dielectric plate

G. Chakraborty
(R. K. M. Vidyamandira, Belurmath, Howrah, W. B., India)

ABSTRACT : The object of the present note is to solve the piezoelectric analogue of the well-known classical problem of pure bending of an electrostrictive dielectric plate with the help of equations of mechanical equilibrium, and those of electricity along with the constitutive equations for an electrostrictive material.

VOL. 20, NO. 1, 1972

Note on the buckling of a thin circular plate of variable thickness under uniform compression

Smarajit Roy
(Visva-Bharati University, Santiniketan, W. B., India)

ABSTRACT : Stability criterion and critical loads in case of buckling of a circular annular plate of variable thickness under uniform compression has been investigated in this paper. The thickness of the plate varies as second power of radial distance from the centre. The inner boundary of the plate is clamped and its outer boundary is clamped and supported.

VOL. 20, NO. 1, 1972

Thermal stress and displacement in a thin rod of finite length in which a steady electric current is flowing, with one end at constant temperature, the other end being fixed and thermally insulated

Snehanshu Kumar Roy Choudhuri
(Jadavpur University, Calcutta, India)

ABSTRACT : The present paper is concerned with the determination of thermo-elastic stress and displacement in a thin rod of finite length heated by an electric current, with one end at constant temperature, the other end being fixed and thermally insulated.

VOL. 20, NO. 1, 1972

Bending of light and its verification

K. C. Kar and A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In a recent paper (Ind. Jour. Theo. Phys. Vol. 20, p. 1, 1972) we have shown that Einstein's relativistic formula for planatary motion, can be derived from simple theory of relativity without the help of his general theory and without

assuming curvature in space-time continuum. The formula thus derived leads to the formula of bending of light on putting the rest mass $m_0=0$. Although in the proofs given by us and by Einstein the mathematical steps are more or less the same, the physical backgrounds are widely different. In Einstein's theory light is bent due to space-time curvature. In the present theory, it is bent because the annihilated chains of electro-positrons through which the electromagnetic wave travels are bent in the gravitational field. Considering the transverse vibration of annihilated chains of electro-positrons under gravity a general formula for bending has been deduced which reduces to Einstein's formula for monochromatic radiation.

VOL. 20, NO. 2, 1972

Note on dynamics of a fluid film of variable viscosity under variable surface charges

Subhash Nandy
(Asutosh College, Calcutta, India)

ABSTRACT : The equations of electricity and the equations of hydrodynamics have been used to solve the problem of flow of an oil film acted upon by an electric field.

VOL. 20, NO. 2, 1972

Large deflection of shallow spherical shells subjected to variable normal load

Amarendra Das
(Jalpaiguri Govt. Engineering College, Jalpaiguri, W. B., India)

ABSTRACT : The paper considers the deflection of shallow spherical shell loaded by variable normal pressure on the concave face. The finite axisymmetric deflections of a thin shell with clamped edges are obtained.

VOL. 20, NO. 2, 1972

Non-steady flow of a conducting Bingham fluid in an annulus in presence of time-varying toroidal pressure gradient

P. N. Tandon

(Harcourt Butler Technological Institute, Kanpur, India)

and

M. D. Raisinghania

(V. S. S. D. College, Kanpur, India)

ABSTRACT : Unsteady flow of a conducting Bingham fluid in an annulus has been discussed in this paper. It has been assumed that a constant radial magnetic field is acting on the fluid and no applied or polarisation voltages exist, *i. e.*, no energy is being added or extracted by electrical means. It has been observed that the toroidal component of the velocity in Bingham fluid is greater than that of viscous fluid and it increases as the value of non-dimensional Bingham number increases.

VOL. 20, NO. 2, 1972

A short note on an approximate form of H-function for isotropic scattering IV

S. Karanjai

(University of North Bengal, Darjeeling, W. B., India)

ABSTRACT : An approximate form for the H-function for isotropic scattering has been discussed. This approximate form is useful in the rough estimation of the line contour.

VOL. 20, NO. 3, 1972

A note on magneto-elastic disturbances of a finite elastic bar of variable cross-section

T. Sengupta

(Syamsundar College, Burdwan, W. B., India)

ABSTRACT : The electro-magnetic equations of Maxwell and the equations of elasticity have been used to investigate the disturbances of an elastic bar acted upon by a magnetic field.

VOL. 20, NO. 3, 1972

On the disturbances in a piezo-electric slab

Shyamal Kumar Chatterjee
(Jadavpur University, Calcutta, India)

ABSTRACT : The mechanical displacement in a piezo-electric slab sandwiched between an elastic and a visco-elastic material, has been investigated in this note. The principle of Laplace transform helps in the solution of the problem.

VOL. 20, NO. 3, 1972

On longitudinal disturbances of an elastic bar in a magnetic field

Ajoy Roy Choudhury
(Karimganj College, Assam, India)

ABSTRACT : The method of transform calculus, the equations of elasticity, electro-magnetic equations of Maxwell have been made use of to investigate the longitudinal disturbances of a partly inhomogeneous, partly homogeneous elastic bar permeated by a magnetic field.

VOL. 20, NO. 3, 1972

Aberration elimination in plane grating spectrographs and monochromators

Mahipal Singh
(Indian Institute of Technology, New-Delhi, India)

ABSTRACT : In this article the author has shown that the aberrations *viz.* astigmatism, coma and curvature of the spectral lines are reduced considerably in convergent and divergent illuminations by taking the forms of the grooves, circular.

VOL. 20, NO. 4, 1972

A note on disturbances in a semi-infinite piezo-electric medium due to a time decaying electric field and time dependent elastic compliance of the medium

Ajoy Roy Choudhury
(Karimganj College, Assam, India)

ABSTRACT : The method of transform calculus is made use of to solve the problem stated in the title.

VOL. 20, NO. 4, 1972

Stresses in a twisted non-homogeneous circular cylinder

Yudhisthir De
(R. K. Mission College, 24-Parganas, W. B., India)

ABSTRACT : Stresses and displacements in a non-homogeneous circular cylinder twisted by shearing forces applied over a region of one plane end, the other end being kept fixed have been obtained in this paper.

VOL. 20, NO. 4, 1972

On transforms and their application

K. D. Sharma
(Govt. College, Churu, Rajasthan)

ABSTRACT : A technipue for defining transforms whose kernels are orthogonal polynomials has been given. Various known transforms are deduced from the general formulæ obtained. Some transforms whose kernels are quite different from those defined as yet have been given. Application of Laguerre transform in finding small transverse oscillations of a very long heavy chain suspended vertically has also been given.

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1973

Relativity in the material world and generalised Lorentz equation

K. C. Kar and A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : It is wellknown that Einstein's relativity is applicable only to vacuum. In section A of this paper the theory of relativity in a medium at rest has been developed by taking velocity of light in the medium as the fundamental velocity. Lorentz's electro-magnetic equation as modified for the medium and Maxwell's other electro-magnetic equations for this medium have been shown to be relativistically invariant, with the help of newly derived transformation relations.

In section B Lorentz equation in vacuum has been derived by a new method. By this method modified Lorentz equations in non-conducting and conducting media have also been obtained. Invariance of Lorentz modified equation in non-conducting medium has been shown in section A. The generalised Lorentz equation in a conducting medium is not invariant since the energy is not conserved in this case.

VOL. 21, NO. 1, 1973

Draining of a powerlaw liquid down a vertical plate in the presence of a transverse magnetic field

Dipak Kumar Dutta
(Jadavpur University, Calcutta, India)

ABSTRACT : The problem considers the motion of a powerlaw liquid under a magnetic field. An analytical method has been developed to discuss the solutions for velocity profile and the rate of flow. From the discussion it is found that the effect of magnetic field damps the motion, whatever be the type of

powerlaw liquid we consider. The problem has its importance in Chemical Engineering—in connection with evaporation, gas absorption experiments, application of coatings to paper rolls and wetted wall towers.

VOL. 21, NO. 1, 1973

Whistler mode propagation in the ionosphere in the presence of a longitudinal static electric field

D. C. Agarwal
(University of Allahabad, Allahabad, W. B., India)

ABSTRACT : The effects due to the longitudinal electrostatic field on whistler mode propagation has been studied in a region of ionosphere where influence of particle collisions and ion moticn may be singnificant. It is found that weak static electric field $\vec{E}_0$ tends to reduce the attenuation of forward whistler when the magnetostatic field $\vec{B}_0$ and the electrostatic field $\vec{E}_0$ are in the same direction. On the other hand when $\vec{E}_0$ and $\vec{B}_0$ are in opposite directions an increase in $\vec{E}_0$ tends to increase the attenuation of the wave.

VOL. 21, NO. 1, 1973

A note on the thermal bending of a solid elastic circular plate with fixed edge due to a concentrated heat nucleus at the centre of the upper face, the lower face being thermally insulated

J. C. Misra
(Uluberia College, Howrah, W. B., India)

ABSTRACT : This note considers the bending of a heated solid circular plate of elastic material due to a nucleus of heat concentrated at the centre of the upper face of the plate. The lower surface is taken to be thermally insulated and the edge of the plate is supppsed to be fixed.

VOL. 21, NO. 1, 1973

Cerenkov radiation from disturbed ionosphere

D. C. Agarwal
(University of Allahabad, Allahabad, W. B., India)
and
S. K. Jain
(Govt. Engineering College, Rewa, India)

ABSTRACT : The paper deals with the cerenkov radiation from abnormally ionized atmosphere. The cerenkov radiated power in the vlf and elf band at mid latitudes is calculated for three disturbed conditions (PCA event, solar flares and nuclear explosion) and ambient condition of the ionosphere. It is found that due to enhanced ion density during disturbed periods more cerenkov power is radiated in these frequency bands.

VOL. 21, NO. 2, 1973

On the mechanical response in a semi-conducting transducer

Ashis Kumar Pal
(Jadavpur University, Calcutta, India)

ABSTRACT : The mechanical response of a semi-conducting piezo-electric transducer have been evaluated under certain prescribed mechanical conditions. The solution of the problem is facilitated resorting to the methods of transform calculus.

VOL. 21, NO. 2, 1973

Propagation of explosion wave in a tube

S. K. Ghosh and Debadideb Bhattacharya
(Jadavpur University, Calcutta, India)

ABSTRACT : The dynamics of propagation of pulses caused by explosion in a tube of finite length has been worked out following powerful operational method due to Heaviside[1]. The problem is of one dimensional in which the explosion is due to an excess pressure P_o within a length $2a$ in a long tude and suddenly released. Pressure variation with respect to time is observed out side the tube length $2a$.

VOL. 21, NO. 2, 1973

Torsional vibration of a thin visco-elastic rod of finite length

Kanchanlal Majumdar
(164, Jodhpur Park, Calcutta, India)

ABSTRACT: This paper is concerned with the torsional vibration of a thin finite rod of Voigt-type visco-elastic material with prescribed twist or shearing stress at one end, the other end being fixed. The solution is expected to throw light on the twisting of some visco-elastic polymers.

VOL. 21, NO. 2, 1973

Mechanical stress response in a piezo-electric plate-transducer corresponding to voltage imput

S. K. Ghosh and Debadideb Bhattacharya
(Jadavpur University, Calcutta, India)

ABSTRACT: The mechanical response of a piezo-electric plate transducer vibrating in a thickness mode has been worked out following the powerful operational method due to Heaviside (1966). The problem is discussed only the case when to one end of the transducer there is a resistive loading. The analysis commences with fundamental piezo-electric equations and solutions are found which represent successive time delayed reflections between the end faces of the transducer.

VOL. 21, NO. 3, 1973

Propagation of love waves in a heterogeneous layer resting on different types of bases

Subhas Dutta
(Bangabasi College, Calcutta, India)

ABSTRACT: Propagation of love waves in a heterogeneous layer in which the rigidity and density obey the law $\mu=\mu_0(1+bz)^n$ and $\rho=\rho_0(1+bz)^n$ resting on different types of bases

and a case in which the layer is sandwitched between two semi infinite homogeneous medium are considered. Frequency equations are obtained in each case and numerical works are performed in some cases.

VOL. 21, NO. 3, 1973

Stability of a low pressure plasma

D. C. Agarwal
(University of Allahabad, Allahabad, W. B., India)

ABSTRACT : The paper presents the analysis of a plasma the pressure of which is small compared with that of the magnetic field. The problem of the equilibrium and the stability of such a plasma is also considered.

VOL. 21, NO. 3, 1973

A note on vibration of a piezo-electric flexural vibrator

S. Mukherjee
(R. K. Mission Vidyamandira, Howrah, W. B., India)

ABSTRACT : In this paper, the frequency of a piezo-electric flexural vibration of a visco-elastic cantilever has been obtained and the results have been discussed.

VOL. 21, NO. 3, 1973

Ion distribution in martian atmosphere

D. C. Agarwal
(University of Allahabad, Allahabad, W. B., India)

ABSTRACT : The paper deals with the study of ion distribution in the atmosphere of Mars. All the calculations have been done for all the three models, namely F_2, F_1 and E. Considering photochemical reactions appropriate to the atmosphere of Mars the positive and negative ion distributions

are calculated. It is found that the upper atmosphere of Mars is dominated by O^+ ions whereas the lower atmosphere is by CO_2^+ ions. Also, it is found that the number density of negative ions is very small.

VOL. 21, NO. 4, 1973

On unsteady flow of a visco-elastic fluid with impulsive pressure gradient acting at any point of a section

Dipak Kumar Dutta
(35, Ruby Park, 24-Parganas, W. B., India)

ABSTRACT : In this note it is intended to solve the problem when a linear visco-elastic fluid flows through a rectangular and a circular duct. The pressure gradient is assumed to be an impulsive function acting at any point of a section of a rectangular channel and also operating on a concentric circular section. Results for the flow of Maxwell fluid and ordinary viscous fluid have been deduced as particular cases.

VOL. 21, NO. 4, 1973

Study of the dislocation damping in solid

Debadideb Bhattacharya
(Rastraguru Surendranath College, 24-Parganas, W. B., India)

and

S. K. Ghosh
(Jadavpur University, Calcutta, India)

ABSTRACT : A quantitative theory of dynamics of dislocation is worked out by the powerful operational method developed by Heaviside (1966). The problem is based on the vibrating string model of dislocation. The attenuation resulting from dislocation motion and viscous damping is calculated in two different cases :— (*a*) When frequency is large. (*b*) When frequency is low. Again the strain-amplitude dependent at low frequencies has been worked out.

VOL. 21, NO. 4 1973

Note on the deflection of a visco-elastic rectangular plate of Reiss type due to an impulsive load in the presence of a transverse magnetic field

Bani Pramanik
(Bethune College, Calcutta, India)

ABSTRACT : In this note the deflection of a linear visco-elastic plate due to a concentrated impulsive load applied at a point on the plate in presence of a transverse, uniform magnetic field has been obtained. The material considered is of Reiss type and the plate is a rectangular one.

VOL. 21, NO. 4, 1973

Limitations in the theory of Relativity

K. C. Kar
(Institute of Theoretical Physics, Calcutta, India)

VOL. 21, NO. 4. 1973

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1974

A new theory of induced disintegration of nuclei

A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : The extension of the new theory of spontaneous alpha disintegration in the present paper[1] to the problem of nuclear reaction has yielded a result giving a new formula for the formation cross-section of the compound nucleus when the target nucleus is bombarded by nucleons. The conception regarding the short range extra nuclear attractive field has been introduced to explain the nuclear reactions when the energy of the incident nucleons (charged) is less than the peak value of coulomb potential. A new formula for formation cross-section

has been derived for these nucleons, agreeing satisfactorily with the experimental results. The incidence of the charged particles having incident energy greater than the coulomb peak and of the uncharged particles have been discussed as special cases.

VOL. 22, NO. 1, 1974

Flow behaviour of blood type suspensions in circular tubes

P. N. Tandon and V. K. Kapoor

H. B. Technological Institute, Kanpur, India

ABSTRACT : Flow behaviour of blood type suspensions has been considered in a large circular tube with high shear rates. Under these circumstances, the viscosities of different shearing laminæ do not depend on the radius of the tube. Approximating the flow to that of several immiscible fluids, the flux across a particular section, and the uniqueness of the velocity maximum have been examined. For the particular case of only two suspensions, it has been observed that the curve of total flux against the ratio of their radii has always a point of inflexion. This ratio has been determined for various values of the ratios of their viscosities when the flux of the two fluids are equal.

VOL. 22, NO. 1, 1974

Study on a two dimensional thermoelastic problem in the presence of a periodic heat nucleus

Bimalendu Das

(Jadavpur University, Calcutta, India)

ABSTRACT : This paper is concerned with the determination of the distribution of displacements and stresses in an infinite elastic solid when the time dependent temperature act upon certain region of the solid. At first a general discussion of two dimensional thermoelastic problem has been made and in the second case displacements and stresses have been calculated due to an insertion of heat nucleus. Strains are taken to be small.

VOL. 22, NO. 1, 1974

Note on the perihelion advance of planets

K. C. Kar and A. K. Bhattacharyya
(Institute of Theoretical Physics, Calcutta, India)

ABSTRACT : In the present paper the problem of planetary motion in a central field has been studied by Lgarange's method. It is shown that the classical approach gives the classical formula for this problem, while the simple relativistic approach (approximate or rigorous) gives the wellknown Einstein formula for perihelion advance (*l,c.*).

VOL. 22, NO. 1, 1974

Radiation from a loop antenna in compressible plasma

D. C. Agarwal
(University of Allahabad, W. B., India)

ABSTRACT : The paper deals with the radiation from a loop antenna in compressible plasma. Expressions for radiation resistance due to plasma wave radiation and electromagnetic wave radiation are derived and they are calculated as a function of the ratio of ω_p/ω.

VOL. 22, NO. 2, 1974

A note on magnetohydrodynamic disturbances in an estuary

S. Basu Mallik
(Jadavpur University, Calcutta, India)

ABSTRACT : In the present note, a theoretical analysis has been carried out for determining the disturbances in an estuary which meets an open sea. The treatment makes use of the electromagnetic equations of Maxwell modified by Lorentz's body force due to the electromagnetic fields induced in the disturbances by their movements through the magnetic field of the earth. Some necessary but simplifying assumptions have been made and it is found that the operational methods facilitate the solution of the problems.

VOL. 22, NO. 2, 1974

A note on the effect of initial stress on SH-wave propagation in a piezo-electric medium

D. K. Sinha

(Jadavpur University, Calcutta, India)

ABSTRACT : As the title of this note implies, it seeks to investigate, using the appropriate equations of SH-motion for a piezo-electric medium, the influence of initial stress on the responses emitted by the medium.

VOL. 22, NO. 2, 1974

On the calculations of noncoherent line contours

S. Karanjai

(University of North Bengal, W. B., India)

ABSTRACT : The equation of transfer for the case of noncoherent scattering has been solved with Giovanelli's form of $\bar{J}(t)$. Disagreement of results calculated with this solution with the observations lead us to consider the form of $\bar{J}(t)$ given by Das Gupta. Calcutations show that by suitably adjusting the constant γ, introduced in Das Gupta's form, the line contour may be made to agree with the observation.

VOL. 22, NO. 2, 1974

Approximate atomic weights of unstable and superheavy elements

B. J. Bhattacharjee

(St. Anthonys College, Shillong, Meghalaya)

VOL. 22, NO. 2, 1974

Analysis of invariance of Maxwell's equations with respect to assymmetrical linear transformations of Voigt-Palacios-Gordon type

M. F. Podlaha

(Institute of Philosophy, West Germany)

ABSTRACT : The invariance of Maxwell's equations with respect to the Voigt-Palacios-Gordon transformations (VPGT)

which admit the transversal contraction of lengths is shown. Poincaré's derivation of the transformation formulæ for E, H and ξ is analysed and a mistake in his derivation is pointed out. It is further shown that the Einstein transformation formulæ for velocities follow from the requirement of invariance of Maxwell's equations with respect to the VPGT and therefore they hold out of the framework of the relativity theory also.

The VPGT contracts into the Lorentz transformation for $\gamma \rightarrow -\frac{1}{2}$. The task of verifying the Lorentz transformation experimentally is therefore to measure the constant γ. This is, however, possible neither by means of experiments of Thorndyke-Kennedy type nor by means of experiments of Trouton-Noble type. γ was first measured by Ives and Stilwell in 1938.

VOL. 22, NO. 3, 1974

Negative ion effect on the absorption of vlf and elf waves in whistler mode

D. C. Agarwal
(University of Allahabad, Allahabad, India)

ABSTRACT : In this paper the effects due to presence of negative ions on the absorption of vlf and elf waves in the whistler mode propagation at midlatitudes are considered for normal days. It is found that negative ions cause an increase in the absorption of elf waves.

VOL. 22, NO. 3, 1974

Radial vibration of an inhomogeneous transversely isotropic cylindrical shell with variable density

Manju Pan
(F-7, Vidyasagar Niketan, Calcutta, India)

ABSTRACT : In this paper the problem of radial vibration of an inhomogeneous transversely isotropic cylindrical shell with variable density has been considered.

VOL. 22, NO. 3, 1974

Deflection of a viscoelastic bar resting on elastic foundation

Madan Mohan Garai
(242/3A, A. P. C. Road, Calcutta, India)

ABSTRACT : In this paper the deflection of a viscoelastic bar resting on elastic foundation has been considered. Two cases have been discussed. The first case is concerned with the deflection due to a concentrated load acting on an infinite viscoelastic bar resting on an elastic foundation and the second one deals with the deflection of a finite viscoelastic bar due to a concentrated load, the bar lying on an elastic foundation.

VOL. 22, NO. 3, 1974

Measurement of the height of the green line 5577 Å emitting layer of the night air-glow at Dumka (Lat : 24° 16′N, long 87°15′ E)

Jogendar Singh
(S. P. College, Bihar, India)
and
S. D. Chatterjee
(Indian Association for the Cultivation of Science, Calcutta, India)

ABSTRACT : Utilising Van Rhijn's method of formulation, the height of the green line λ 5577 Å, emitting layer of the night air-glow has been experimentally determined at Dumka (lat : 24°16′N ; long : 87 °15′E). The measured height of 174 km. may be taken as the optical centre or the centre of gravity of two emitting layers, one located near 100 km. and the other near 250 km.

VOL. 22, NO. 4, 1974

Effect of contact resistance for the linear flow of heat in an infinite solid

Pradip Kumar Sen
(Mahadevananda Mahavidyalaya, 24-Parganas, W. B., India)

ABSTRACT : In order to consider the effect of contact resistance on the heat conduction, a problem of linear flow of

heat in an infinite composite solid has been worked out by Laplace transformation method. The method of seeking solutions appropriate to a particular problem is not an easy process ; but by the present method the correct solution is obtained in a very simple way and a special case is also worked out from the general theory. Both for general and special cases the expressions for temperature distribution in the different regions of the solid are obtained and some important conclusions are drawn from the graphs of temperature vs. distance and temperature vs. coefficient of surface heat transfer.

VOL. 22, NO. 4, 1974

Strain wave in a linear array of oscillators

R. Manvi

(California State University, California, U.S.A.)

ABSTRACT : The system analyzed in this paper consists of an infinite array of mass points with linear nearest neighbour interaction. There is no dissipation in the system. Initially, we have uniform strain conditions on the left hand side of the array and zero strain on the right hand side. At zerotime, the particles are released. This problem is analogous to the physical case of a shock tube in gas dynamics. It is shown that even though the subsequent motion is oscillatory, but for very large times uniform conditions result again in the array.

VOL. 22, NO. 4, 1974

Dynamics of the vibration of a built-in bar excited by transverse impact of a hard load at the centre of the bar

K. Ray and R. K. Chatterjee

(Research & Contact Laboratory, Durgapur, W. B., India)

ABSTRACT : This paper deals with the dynamics of vibration of a bar fixed at both ends and excited by transverse impact of a hard hammer at the midpoint. It has been assumed that the

bar behaves as a loaded bar so long as the hammer is in contact with the bar. Expressions for the displacement at the struck point during impact,, pressure exerted by the hammer during impact have been obtained by following the operational method.

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Ionospheric phase distortion at UHF band

S. K. Jain, D. C. Agarwal and M. B. Jain

(Govt. Engineering College, Rewa, M.P., India)

ABSTRACT: The paper deals with the calculations of ionospheric phase dispersion which is made up of the phase path delay and group path delay, in the wide band (100 *MHz*) communication at *uhf* band (790-890 *MHz*), suffered by the ratio waves while propagating through ionosphere. It is found that the phase dispersion is maximum during equinox day time.

VOL. 23, NO. 1, 1975

Radial flow of heat in two concentric spheres having a contact resistance

Pradip Kumar Sen

(Mahadevananda Mahavidyalaya, 24-Parganas. W.B., India)

ABSTRACT: The paper is concerned with the radial flow of heat in two concentric spheres in presence of contact resistance with a steady source of heat. Heaviside's operational method[6] is used to solve the problem. The importance of this paper is that the different effects of contact resistance on the temperature distribution for radial heat flow are discussed here in some special cases.

VOL. 23, NO, 1, 1975

Vibrations of an inhomogeneous spherical shell of aeolotropic material with variable density

Manju Pan
(F-7, Vidyasagar Niketan, Calcutta, India)

ABSTRACT : Radial and rotatory vibrations of an inhomogeneous spherical shell of aeolotropic material with variable density have been considered in this paper.

VOL. 23, NO. 1, 1975

Relativistic mass of a free particle and a particle in dynamical equilibrium

K. C. Kar
(Institute of Theoretical Physics, Calcutta India)

ABSTRACT : The well-known relativistic formula connecting the mass of a moving particle in S and S' systems is $m=m_o\beta$. In evaluating the relativistic kinetic energy Einstein has used the formula $m=m_o\beta^3$ naming it the longitudinal mass of the particle. The mass given by the β-formula has been called by him the transverse mass. This nomenclature of Einstein has for many years misled relativists to think that the transverse mass of a particle is its mass measured from a direction transverse to the direction of motion of the particle while the longitudinal mass is its mass measured from an S-system placed in the direction of motion of the particle. After the proof given by the writer that the mass is a contravariant vector in the direction of time and so all directions in space are transverse to it, it is not proper to distinguish between transverse and longitudinal mass after Einstein. In the present paper it is shown mathematically that the limiting conditions of energy are satisfied not by β-formula but only by β^3-formula taken by Einstein. It is also shown purely on physical ground that the β-formula is to be taken whenever the particle is free while the β^3-formula is to be taken when the particle is bound in a dynamical system as assumed in evaluating the kinetic energy. It is to be noted that during motion the planets are not free but are in dynamical equilibrium. Therefore, in these cases

β^3-formula for mass should be used. It is proved that on using this β^3-formula one readily gets Einstein's three general relativity formulæ without any help of his general theory of relativity. So, all the general relativity effects are explained from the simple theory of relativity.

VOL. 23, NO. 1, 1975

Propagation of waves in hot plasma waveguides

Ramesh Chandra Soni and P. V. Bakore
(Govt. College, Shahpura (Raj), India)

ABSTRACT : Taking collisions into account the problem of wave propagation in the axial direction of a plasma filled circular waveguide has been studied through the use of Boltzmann equation and Maxwell's field equations following Kuehl. The dispersion equation has been discussed for particular cases of interest. It has been observed that the collision frequency ν plays a significant role in the damping of low-frequency waves at the boundary wall of the waveguide.

VOL. 23, NO. 2, 1975

On the Coulomb interactions

M. Dhara
(Jadavpur University, Calcutta, India)

ABSTRACT : The behaviour of two non-relativistic charged particles interacting via a short-range separable potential and a repulsive Coulomb potential is studied. Using one term of the S-wave separable potential of the Yamaguchi type and the scattering lengths of proton-proton, neutron-neutron and neutron-proton, the corresponding coupling constants are estimated.

VOL. 23, NO. 2, 1975

A note on vibration of a piezo-electric crystal with dissipative mechanical parameters

S. N. Ganguly
(Hindu College, 24-Parganas, W. B., India)

ABSTRACT : This paper is an attempt to investigate the vibration in a piezo-electric crystalline medium whose mechanical

parameters are characterised by dissipative features. In particular, the vibrations for a x-cut quartz crystal have been determined.

VOL. 23, NO. 2, 1975

Length contraction and time dilatation in the special theory of relativity-real or apparent phenomena?

M. F. Podlaha
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ABSTRACT : The relativistic statement that the shortening of moving bodies and retardation of moving clocks are only apparent phenomena is discussed. The notions "absolute", "real", "relative" and "apparent" are reexamined and redefined. By means of the Voigt-Palacios-Gordon transformation and two theorems, it is shown that there exists one co-ordinate frame S with respect to which the length contraction and time dilatation are real-absolute phenomena, in spite of the fact that the frame S cannot be experimentally distinguished from the frames S', S''...moving uniformly in a straight line direction with respect to S. It is further shown, that the length contraction and time dilatation observed in the frames S', S'', although they are relative phenomena, cannot be viewed as a consequence of a "perspective of observation" only, as it is often erroneously stated by relativists, The results of the relativistic experiments have thus their origin in the effects which are so real as a change of the length of a rod caused by the change of the temperature.

VOL. 23, NO, 2, 1975

Reflection and transmission of electromagnetic waves by a moving compressible plasma slab

D. C. Agarwal
(University of Allahabad, Allahabad, India)

ABSTRACT : The paper deals with the reflection and transmission of electromagnetic waves by a moving compressible plasma slab. The power reflection co-efficient for the moving

cold plasma slab and that for the moving compressible plasma case has been calculated and compared with each other. It is found that because of the compressibility of the plasma the power reflection co-efficient becomes oscillatory in nature.

VOL. 23, NO. 2, 1975

Longitudinal wave propagation in warm electro-magneto-plasma

Surendra Rawat
(Government College, Jaipur, India)

ABSTRACT : The dispersion relation for the longitudinal waves propagating in a drifting, warm, collisional electron plasma subjected to the parallel electrostatic and magnetostatic fields has been worked out and the complex propagation constant and the $\partial\omega/\partial K$ have been investigated in detail for some regions of the ionosphere. It is observed that at higher regions of the ionosphere $\partial\omega/\partial K$ is a real quantity equal to the drift velocity, which is the result of the presence of electrostatic field in the ionosphere otherwise $\partial\omega/\partial K$ is a complex quantity. If $\delta<1$ or $\delta\geqslant 1$ and $\nu=2\omega_p$ the $\partial\omega/\partial K$ becomes real even for the evanescent longitudinal waves.

VOL. 23, NO. 3, 1975

Sound transmission through plates-application of the method of resolution of acoustic waves

M. C. Bhattacharya
(Jalpaiguri Govt. Engineering College, W. B., India)

ABSTRACT : This paper presents a theory by which the pressure gradient of a propagating acoustic wave can be resolved into two components propagating at right angles to each other. Such resolution of acoustic waves and its application to the formulation of the mass law of sound transmission have not been discussed earlier. It would appear from this paper that this method of analysis can be very successfully used in the study of vibration problems induced by obliquely incident acoustic waves and in many other acoustical problems.

The method of analysis, hitherto in use, for the study of sound transmission through plates cannot, perhaps, adequately and accurately take into account the effect of the angle of incidence of the acoustic wave. The method of resolution of acoustic pressure gradient into components enables a more accurate and satisfactory evaluation of an expression for the sound transmission loss due to an infinite plate. It also explains the increased sound transmission at higher angles of incidence. The theory could also be applied in the study of the coincidence effect associated with the transmission of sound energy through plates.

This analysis involving resolution of the acoustic pressure gradient into components is likely to be useful in all fields of acoustics. The present analysis is mainly confined to an ideal infinite plate.

VOL. 23, NO 3, 1975

On the propagation of plane waves in a half space of visco-elastic material of Burger's type subjected to a magnetic field

Mala Mahalanabis

(Jadavpur University, Calcutta, India)

ABSTRACT : This paper deals with the propagation of plane waves in a half-space of visco-elastic material of Burger's type when the medium is excited by a magnetic field. It is solved by making use of the electromagnetic equations of Maxwell, equations of mechanical motion and stress-strain relations of the material.

VOL. 23, NO. 3, 1975

আইনষ্টাইনের সাধারণ আপেক্ষিক তত্ত্বে কয়েকটি গুরুত্বপূর্ণ ত্রুটি

Some serious defects in Einstein's generalised theory of relativity

K. C. Kar

(Institute of Theoretical Physics, Calcutta India)

ABSTRACT : It is shown that Einstein's assumption in taking the time axis inclined to the space co-ordinates in the

gravitational field in his generalised theory of relativity is wrong and has no physical basis. In a recent paper the writer has shown that the attractive gravitational field originates from the infinite number of fine almost annihilated chains of opposite electrical charges filling the ether. Thus the gravitational field is always attractive and is in no way related to the inclination of the time axis with the space co-ordinates, assumed in Einstein's Theory. The laws of attraction are same as in classical theory. Because in simple theory of relativity the mass is dependent on the velocity, Kepler's law of area is to be relativistically corrected, remembering that there is no external transverse force in planetary motion. After this correction and after easy transformation we easily get the corrected equation of motion of Einstein. Thus all the three effects, namely, (1) bending of light in a gravitational field, (2) Perihelion advance of planets and (3) the red-shift are explained as by Einstein. There is no necessity of introducing oblique co-ordinates of tensors and of taking shelter behind non-Euclidean geometry as has been done by Einstein. Accordingly the whole subject of generalised relativity can now be discussed on the basis of the simple theory of relativity.

VOL. 23, NO. 3, 1975

Some speculations on "Antiphotons"

John Di Prete

45, Vale Avenue, Cranston, U. S. A.

VOL. 23, NO. 3, 1975

Heat flow in a semi-infinite composite medium with periodic surface temperature

Pradip Kumar Sen

(Mahadevananda Mahavidyalaya, 24 Parganas W. B., India)

ABSTRACT : The paper is concerned with the heat flow in a semi-infinite composite medium with periodic surface temperature. Heaviside's operational method is used to solve the problem. Expressions for temperature distribution in the

different regions of the composite medium are obtained. Also an important special case, of a thin film attached to an infinite solid, has been worked out from the general theory.

VOL. 23, NO. 4, 1975

Note on stress waves in a viscoelastic rod of general linear type

Satya Narayan Mandal

(Hatuganj M. N. K. High School (Multipurpose)
24-Parganas W. B., India)

ABSTRACT : The object of this note is to find the stress in a semi infinite viscoelastic rod of general linear type when an arbitrary time-dependent transient force is applied at its finite end. Laplace transform and convolution theorem have been used to solve the problem.

VOL. 23, NO. 4, 1975

Magnetohydrodynamic flow of an electrically conducting viscoelastic fluid in a composite slider

P. N. Tandon and S. C. Pokhariyal

(H. B. Technological Institute, Kanpur, India)

ABSTRACT : Presented herein are the studies of the magnetohydrodynamic flow of a conducting visco-elastic fluid in a composite slider. It has been observed that the effect of visco-elasticity is to increase the load carrying capacity and applied magnetic field also increases the load carrying capacity. It has also been observed that the load capacity decreases with the increase in the length of the flat part or the slope of the inclined part.

VOL. 23, NO. 4, 1975

Note on the unsteady flow of viscous incompressible fluid of finite depth over a fixed rigid plane base due to an arbitrary time varying pressure-gradient

Dilip Kumar Das

(North Bengal University, Darjeeling, W. B., India)

ABSTRACT : In this paper, the problem of unidirectional flow of a viscous incompressible fluid under prescribed pressure gradients is solved.

VOL. 23, NO. 4, 1975

On the binding energy of triton

M. Dhara

(Indian Association for the Cultivation of Science, Calcutta, India)

ABSTRACT : Using the Faddeev techniques, the binding energy of triton and the Coulomb energy of ^{3}He have been calculated on the assumption that the interaction between the nucleons is described by a nonlocal potential with separable variables.

VOL. 24, NO. 1, 1976

Transverse oscillation of a viscoelastic bar due to an impulsive load acting at its middle point

Madan Mohan Garai

(134/A, Masjidbari Street, Calcutta, India)

ABSTRACT : Taking into consideration the inertia terms in the equation of bending of a viscoelastic rod, the problems stated in the title have been solved in this paper. Assuming the length of the bar to be 'a', expression for deflection at the centre when $t \to \infty$ has been obtained.

VOL. 24, NO. 1, 1976

A note on propagation of disturbances in a piezo-electric medium under the influence of an external electron stream

Miss Samita Basu

(Jadavpur University, Calcutta, India)

ABSTRACT : The present note is an attempt to investigate some aspects of disturbances in a piezo-electric material under the influence of an electron stream using the equations of electricity and that of mechanics of continua together with the constitutive equations of the material.

VOL. 24, NO. 1, 1976

Instabilities in double phase flow through dipping porous medium with capillary pressure

S. K. Mishra

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ABSTRACT : The present paper analytically discusses the phenomenon of instabilities (fingering) in double phase flow through dipping porous media. A specific oil-water displacement with well developed finger flow in a dipping homogeneous porous medium with capillary pressure, has been analytically investigated by assuming a statistical model. The underlying assumptions made to the present analysis is that the individual pressure of the two flowing phases may be replaced by their common mean pressure. A mathematical solution of the nonlinear differential system governing fingering has been obtained by perturbation method. Finally an analytical expression for the average cross-sectional area, occupied by fingers has been derived and the possibility of finger stabilization is shown under certain specific conditions.

VOL. 24, NO. 1. 1976

Magneto-thermo-elastic disturbances in a semi-infinite bar of variable cross-section and of variable magnetic permeability

N. G. Mookerjee

(Abhedananda Mahavidyalaya, Birbhum W. B. India)

ABSTRACT : The electro-magnetic equations of Maxwell, the equation of mechanical motion and the equation of heat-flow have been used to solve the one-dimensional problem of magneto-thermo-elastic disturbances in a semi-infinite bar of variable cross-section when its magnetic permeability depends upon the elastic strains.

VOL. 24, NO. 1. 1976

On the propagation of space charge waves in an accelerated electron beam

D. C. Agarwal

(J. K. Institute of Applied Physics and Technology
University of Allahabad, Allahabad, India)

ABSTRACT : This paper deals with the studies of propagation of fluctuations in an electron beam. Equations of space charge waves in the case that the potential distribution is independent of the space charge of the beam are deduced. Equations of space charge waves in several special cases are also deduced.

VOL. 24, NO. 2. 1976

Variational approach to the scattering of slow electrons by polar molecules

M. P. Maru

(D. K. V. Arts and Science College, Jamnagar, Gujarat)

and

H. S. Desai

(M. S. University, Baroda, Gujarat)

ABSTRACT : The momentum transfer cross-section for slow electron scattering by polar molecules, is calculated by Schwinger's variation method. The results for different "l" values are compared with Born approximation and partial wave method.

VOL. 24, NO. 2. 1976

Dynamics of the vibration of a built-in bar excited by transverse impact by an elastic load

K. D. Ray

(Abhedananda Mahavidyalaya, Birbhum W. B., India)

and

R. K. Chatterjee

(Alloy Steels Plants, Durgapur, India)

ABSTRACT : This paper deals with the dynamics of vibration of a bar fixed at both ends struck transversely at any point of the bar by an elastic load. It has been assumed that the bar behaves

as a loaded bar so long as the hammer is in contact with the bar and the elastic hammer is conceived of as a perfectly hard hammer backed by a weightless spring. The expressions for the displacement at the struck point during impact, pressure exerted by the hammer during impact have been obtained by following the operational method. The case of a hard hammer dealt with earlier follows as a special case by putting the elastic constant of the hammer equal to infinity.

VOL. 24, NO. 2, 1976

Transient temperature distribution in a non-homogeneous solid cube

K. D. Sharma

(Government Lohia College, Churu, Rajasthan)

ABSTRACT : The transient temperature distribution in a non-homogeneous solid cube is found out with the help of Jacobi transform. The laws of variations of thermal conductivities along the principal axes are taken in such a way that all the six faces of the cube are insulated and hence the flux of heat vanishes on all the six faces. Therefore no boundary conditions are prescribed.

VOL. 24, NO. 2. 1976

Radial and rotatory vibrations of a transversely isotropic non-homogeneous thin annular disk with varying elastic parameters

Mrs. Manju Pan

(F-7, Vidyasagar Niketan, Salt Lake, Calcutta, India.)

ABSTRACT : In this paper the problems of radial and rotatory vibrations of a transversely isotropic non-homogeneous thin circular disk have been considered. The elastic parameters are given by $C_{ij}=a_{ij}\, r^n$, a_{ij} and n being constants and the density is given by $\rho=\rho_0 \cdot r^s$, where ρ_0 and s are constants, r being the distance measured from the centre of the disk.

VOL. 24, NO. 2, 1976

Non-existence measurement and its relationship to time and motion

John Di Prete

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VOL. 24, NO. 2, 1976

The non steady three dimensional flow in the boundary layer on a body executing simple harmonic motion

R. Askovic

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ABSTRACT : The three-dimensional unsteady laminar boundary layer on a body in the periodic motion along a linear trajectory is discussed. First, the solution of this problem is found by the method of successive approximations. Subsequently, applying this solution it is proved that there are both longitudinal as well as transverse components of the induced stationary flow at a sufficiently large distance from an arbitrary oscillating body.

VOL. 24, NO. 3, 1976

Mechanical response of a non-uniform pre-energised semi conductor transducer subjected to a body force and loaded with a delay rod

N. C. Ghosh

(Jadavpur University, Calcutta, India)

ABSTRACT : In the present paper the mechanical response of a pre-energised non-uniform piezoelectric transducer loaded with a delay rod under the presence of a body force has been attempted, for small values of time.

VOL. 24, NO. 3, 1976

Forced torsional vibration of a cone with spherical caps twisted by periodic terminal couples

A. Mukhopadhyay

(Vivekananda Mahavidyalaya, Burdwan, India)

ABSTRACT : Problem of forced torsional vibration of a non-homogeneous cone with spherical caps, where the density as well as the rigidity modulus varies with some power of the distance from the vertex of the cone by applying periodic terminal couples has been solved and the effects of non-homogeneity on the stresses have been exhibited.

VOL. 24, NO. 3, 1976

On the analysis of lens like media using variational method

D. C. Agarwal

(University of Allahabad, Allahabad India)

ABSTRACT : The paper deals with the analysis of lens-like media using the variational method. An expression for the propagation constant for optical waves propagating through such a lens having the dielectric constant which varies in square law as well as fourth power law has been derived.

VOL. 24, NO. 3, 1976

Model potentials for ionic crystals

(Review Paper)

A. K. Sarkar and S. Sengupta

(Presidency College, Calcuttta, India)

ABSTRACT : The effective interaction in ionic crystals have been studied very thoroughly, both from the emperical and theoretical stand point. The purpose of the present review is to present a complete discussion of the upto date situation. At first the two body potentials both short range and long range, are discussed. The inadequacy of these potentials are pointed out. Then the short range three body interactions are discussed. In

this connection a new treatment of Lowdin and Lundquvist's work is given. The works of Jansen et al and exchange charge model is adequately treated. The new three body potential proposed by Sarkar and Sengupta is also discussed. Then the long range three body interaction arising from the quantum mechanical perturbation treatment of the ionic interaction is treated. Finally, the many body interactions in several of the lattice dynamical models currently used in the treatment of ionic crystals are discussed.

VOL. 24, NO. 3, 1976

Temperature variation of the electrical resistivity of lithium, potassium and rubidium

O. P. Gupta

(J. Christian College, Allahabad, India)

ABSTRACT : The temperature variation of the electrical resistivity of lithium, potassium and rubidium is calculated within the free-electron approximation using the anisotropic continuum dispersive model proposed by Sharma and Joshi for the lattice dynamics of metals. The theoretical results are compared with the available X-ray measurements. Except for lithium, there is a reasonable good agreement between the theory and the experimental data.

VOL. 24, NO. 4, 1976

On the linearized Boltzmann's equation for semiconductor plasmas

D C. Agarwal

(University of Allahabad, Allahabad India)

ABSTRACT : This paper deals with some aspects of the Boltzmann equation used in solid state plasmas. The effects of the collision terms on the phase velocity of carrier wave in semiconductors are also discussed.

VOL. 24, NO. 4, 1976

Note on the transverse vibration of an isotropic circular plate with density varying parabolically

Mrs. Manju Pan

(F-7, Vidyasagar Niketan, Salt Lake, Calcutta, India)

ABSTRACT : The object of this note is to solve the problem of transverse vibration of an isotropic circular plate having density varying parabolically.

VOL. 24, NO. 4, 1976

Behaviour of a Bingham fluid in the entry region of a pipe

P. N. Tandon and M. D. Raisinghania

(H. B. Technological Institute, Kanpur, India)

ABSTRACT : Flow of a slurry (Bingham Fluid) in the entry region of a circular pipe has been studied, accounting for the loss of energy due to viscous dissipation in the boundary layer. Pressure has been obtained by the application of macroscopic mechanical energy balance to all the fluids in the entry region. These problems have also been solved by using Schiller's approach for the entrance region and the two results have been compared. It has also been pointed out that the former approach is an improvement over the latter and provides a solution which is more satisfactory for practical applications.

VOL. 24, NO. 4, 1976

Thermal stresses in an infinite elastic medium containing an external crack

S. N. Maiti

(Indian Institute of Technology, Kharagpur, India)

ABSTRACT : A problem on the thermal stresses in an infinite homogeneous isotropic medium containing an external crack has been considered in this paper by using integral transform techniques. Two cases have been considered *viz* (*i*) when the

surfaces of the crack are subject to a prescribed temperature and (*ii*) when the surfaces of the crack are subject to a prescribed heat-flux.

VOL. 24, NO. 4, 1976

Electrical response in a piezo-electric plate transducer under the influence of a body force

Pradip Kumar Sen

(Mahadevananda Mahavidyalaya, Barrackpur-743101, 24-Parganas, W. B., India)

ABSTRACT : The response of a piezo-electric plate transducer vibrating in a thickness made excited by transient stress input has been worked out following the powerful operational method due to Heaviside[3]. The problem is discussed in presence of a body force with two possible conditions, *viz.* (*i*) the transducer is open at both ends and (*ii*) to one end of the transducer there is a resistive loading.

VOL. 25, NO. 1, 1977

Note on the vibration of circular plates of variable thickness

J. C. Kunda

(Prachya Bharati H. S. School, Tripura, India)

and

S. Basuli

(Tripura Engineering College, Tripura, India)

ABSTRACT : Following Rayleigh-Ritz method for vibrating plate problem, the note aims at investigating the problem of vibrations of clamped circular plates of variable thickness. Numerical calculations to the frequencies are tabulated for various values of the thickness.

VOL. 25, NO. 1, 1977

Multi phase flow of incompressible immiscible fluids in an annulus

V. K. Kapoor and R. P. Singh

(H. B. Technological Institute, Kanpur, India)

ABSTRACT : Flow behavior of several incompressible immiscible fluids, with different viscosities, in concentric laminae of different thicknesses has been considered. It has been observed that a unique velocity maximum always exists and a formula for finding the layer in which this velocity maximum occurs has been formulated. This analysis will be of use in understanding multiphase flows which have great importance in chemical engineering.

VOL. 25, NO. 1, 1977

Reflection of electron plasma waves from flat plane boundary

D. C. Agarwal

(Tohoku University, Sendal, Japan)

ABSTRACT : This paper deals with the study of reflection co-efficient of electron plasma waves from a flat plane boundary. Using the boundary conditions derived by hydrodynamical approach and two stream Maxwellian distribution method, an expression for the reflection co-efficient for a rigid as well as for an absorptive layer has been derived.

VOL. 25, NO. 1, 1977

On torsional transient motion of a visco-elastic maxwell liquid between two concentric spheres

T. R. Roy

(Jadavpur University, Calcutta, India)

ABSTRACT : A few solutions exist for problems in liquid flow about a rotating sphere contained in another concentric sphere. Here the liquid under investigation is a visco-elastic Maxwell liquid and we are interested in finding the velocity

component in the ϕ-increasing direction. The couple experienced by the inner sphere is also calculated. The method of solution given by Basset[1] is adopted.

VOL. 25, NO. 1, 1977

Remarks on a reduction scheme for principal value collision integrals

B. R. Wienke

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ABSTRACT : Principal value integrals occurring in non-relativistic perturbation theory can be evaluated by alternatively solving differential equations obtained from the inversion of the T operator equation. Such procedure leads to an interesting representation of perturbation theory.

VOL. 25, NO. 1, 1977

De Broglie waves, length contraction and time dilatation

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VOL. 25, NO. 1, 1977

A note on disturbances in a piezo-electric slab due to a tangential electric field of periodic type under the influence of a body force and when a constant heat flow is maintained.

N. C. Ghosh

(Jadavpur University, Calcutta, India)

ABSTRACT : In this paper the mechanical disturbances produced in a piezo-electric slab under the action of a periodic electric field has been obtained for large values of time when it is under the action of a body force and constant flow of heat. Lastly, the methods of Laplace transform have been used to facilitate the solution of the problem.

VOL. 25, NO. 2, 1977

On mechanical response in a piezo-electric transducer subjected to a current flowing in a semi-conducting boundary layer

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ABSTRACT : The present article is concerned with the investigation of mechanical responses in a piezo-electric transducer acted upon by a current flowing in a semi-conducting boundary layer taking a prescribed diffusion into account. A comparative study of response when the boundary layer is absent has also been made.

VOL. 25, NO. 2, 1977

On the holography by object scanning

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ABSTRACT : The paper deals with the study of holography by scanning the objects. Useful expression for the resolving power for this type of hologram has been derived and it is shown that the resolving power for this case is increased in comparison to that holography by the source and the receiver.

VOL. 25, NO. 2. 1977

Flow of cholesteric liquid crystal between two parallel plates

A. K. Sharma

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ABSTRACT : In the present paper, flow of cholesteric liquid crystal between two parallel plates, one being at rest and the other moving with a uniform velocity along a straight line in its own plane, has been discussed. A solution of the problem has been obtained for small shearing stresses.

VOL. 25, NO. 2, 1977

Drag on a circular cylinder moving with a transient velocity in an infinite expanse of a visco-elastic maxwell liquid

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ABSTRACT : A circular cylinder is moving with a transient velocity in an infinite expanse of a visco-elastic Maxwell liquid along a line $\theta=0$. In this paper the velocity components and the resultant force on the cylinder are calculated.

VOL. 25, NO. 2, 1977

The angular momentum group of four quarks

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VOL. 25, NO. 2, 1977

On the radiation from a magnetic line source on a perfectly conducting plane into a compressible, anisotropic plasma

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ABSTRACT : In this paper an attempt has been made to study the radiation characteristics of a magnetic line source on a perfectly conducting plane into a compressible, anisotropic plasma. Expressions for far field pattern have been derived and discussed.

VOL. 25, NO. 3, 1977

Simultaneous development of momentum and thermal boundary layers in a suddenly started visco-elastic fluid in the entrance region of a circular pipe

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ABSTRACT : The simultaneous developments of momentum and thermal boundary layers in a suddenly started visco-elastic fluid in the entrance region of a circular cylinder is discussed here. Following the integral momentum method, the equations of motion and temperature distribution in the entrance region are transformed into quasi-linear differential equations. The numerical results have been given in tables 1 to 4 and some of them have been plotted graphically. It has been observed that entrance length decreases with the increase of visco-elastic parameter. The entrance length increases with the increase in Reynold's number. The thermal entrance length increases with the increase in Prandtl number.

VOL. 25, NO. 3, 1977

Heat flow in a semi-infinite composite medium with periodic surface temperature having a contact resistance

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ABSTRACT : Heaviside's operational method (1966) has been applied to solve a problem of heat conduction in a semi-infinite composite medium with periodic surface temperature having a contact resistance. Expressions for temperature distribution in the different regions of the composite medium are obtained and also some important special cases are discussed based on the general theory.

VOL. 25, NO. 3, 1977

Buckling of a heated thin annular circular plate of variable thickness

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ABSTRACT : This paper deals with the buckling of a heated annular circular plate of thickness varying as the m-th power of the radial distance ($m<1$). The general stability criterion has been obtained whence the critical buckling compression as well as the critical temperature has been obtained.

VOL. 25, NO. 3, 1977

A note on flow of liquid in a visco-elastic tube

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ABSTRACT : The note aims at investigating the flow of liquid in a tube made up of visco-elastic material. Different aspects of wave propagation have been discussssed.

VOL 25, NO. 3, 1977

A note on stress distribution in a plate under simple tension in electrostriction

S. C. Ghosh

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ABSTRACT : This note is an attempt to determine the nature of stress distribution in a plate with a circular hole under simple tension in electrostriction, based on the field equations derived by Knops[1] for two dimensional problems of electrostriction within the framework of complex variable theory.

VOL. 25, NO. 3, 1977

INDIAN JOURNAL OF THEORETICAL PHYSICS

SILVER JUBILEE VOLUME

SECTION II

INDIAN JOURNAL OF THEORETICAL PHYSICS
SILVER JUBILEE VOLUME (1953-1977)

On the theory of lamellar reflection grating

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(Received for publication in March 1977)

[**Abstract :** In this paper, a theoretical analysis has been presented on the intensity of the diffracted light obtainable by a lamellar reflection grating. The problem has been attempted by examining the effect of the elevations and that of the depressions separately and thus the expression for the resultant intensity has been derived. It has been found that the lamellar reflection grating is equivalent to a combination of a plane grating and an echelon grating.]

Introduction

It is well known that gratings are preferred to prisms in high resolution work near infra red. In the transmission type of grating, the energy is not concentrated to a particular spectrum and as a result, the intensity of the spectra becomes weak. This difficulty has been overcome by echelle and laminary gratings, which if suitably designed are capable of concentrating the energy in the first order. The use of laminary grating was first studied by Quincke.[1] He simply replaced the opaque strip of a plane grating by transparent strips of such thickness that some one wavelength suffers a retardation of $\frac{\lambda}{2}$. If light of this specified wavelength is used, the grating will fail to show the central image. On the other hand if white light is employed, the central image instead of being white appears coloured.

A considerable amount of work has been done both on theoretical as well as experimental side on echellette grating[2,3] which was first ruled by Wood[4,5,6]. But the laminary or lamellar reflection grating[7], which appears to have promising

role in spectroscopy has not been so widely investigated though such gratings were developed by Cartwright[8] and Hellwege[9] as early as 1931 and 1937 respectively. It is only very recently that such gratings have attracted the attention of the spectroscopists after the development of a new method of its construction principally at the National Physical Laboratory by Sayce and Franks[10] and others[11] with which it is possible to investigate the diffraction upto *nm* range.[12] In this context, it was considered necessary to analyse its theory in some more details specially in regard to the intensity distribution of the diffracted light by such gratings and this paper is a modest attempt towards this end.

Construction of lamellar grating : The lamellar reflection grating is also called the rectangular laminary profile. To form such gratings, a metal is deposited on a plane plate by evaporation. The deposited metal is then partially removed from alternate spaces between equidistant parallel lines on the surface of the deposit producing rectangular hollow grooves as shown in Fig. 1. Such gratings have been constructed now a days by new methods as mentioned earlier.[9,10]

Theory of lamellar grating : Let us consider the Fig. 1. Let d be the depth of the grooves. Let the breadths of alternate

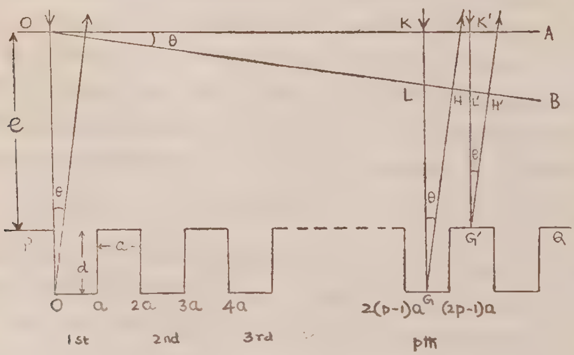

Fig. 1

elevations and depressions be each equal to a. One element of the grating consists of one elevation and a depression next to it,

the magnitude of the grating element thus being $(a+a)$. Such grating are generally used for normal incidence of rays.

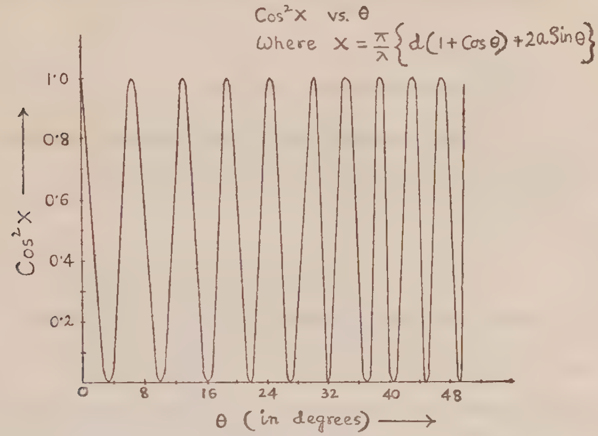

Fig. 2

The entire grating surface may be supposed to be divided into two parts :

(*i*) that consisting of elevations only ; this we shall call the E grating.

(*ii*) that consisting of depressions only ; this we shall call D grating.

Let us consider rays diffracted at angle θ from each point of incidence with respect to the normally incident rays. In Fig. 1, OA is the incident wave front, OB is the diffracted plane wave front inclined at an angle θ to OA. All rays diffracted at angle θ are brought to focus at F (not shown in the figure) by means of a converging lens. All optical paths from the diffracted wave front OB to the focus F are necessarily equal.

The D-type grating : The path difference between the incident and diffracted wave fronts from a point G in the *pth.* depression is

$$\begin{aligned}\Delta_p D = KG + GH &= KG + GL \cos\theta \\ &= KG + (KG - KL)\cos\theta \\ &= KG(1+\cos\theta) - OK \tan\theta \cos\theta \\ &= KG(1+\cos\theta) - OK \sin\theta.\end{aligned}$$

Taking the x-axis along the direction OA with origin at O and taking $OK=x$, $KG=e+d=$ constant we have,

$$\Delta pD=(e+d)(1+\cos\theta)-x\sin\theta$$
$$=\alpha+\beta x \qquad \cdots \qquad (1)$$

putting $(e+d)(1+\cos\theta)=\alpha$ and $-\sin\theta=\beta$.

Let the equation of vibration on the incident wave front OA (taking amplitude equal to unity) be,

$$y_0=\exp\{ikct\} \qquad \cdots \qquad (2)$$

where $k=\dfrac{2\pi}{\lambda}$ and $i=\sqrt{-1}$ and $c=$ velocity of light.

Then the equation of vibration at the point H is given by,

$$y_H=\exp[ik(ct-\Delta pD)] \qquad \cdots \qquad (3)$$

The displacement at the focus F due to secondary waves coming from a small element dx of the incident wave front at K is given by,

$$dS_{pD}=\exp\{ik(ct-\Delta pD)\}dx$$
$$=\exp\{ik(ct-\alpha)\}\exp(-ik\beta x)dx \qquad \cdots \qquad (4)$$

The total displacement at the focus F due to all elements of the D-grating is to obtain by integration over all depressions of D-grating. The limits of integration extend from 0 to a, $2a$ to $3a$, $4a$ to $5a$ etc. for the first, second, third etc. depressions. For the pth depression the integration extends from $2(p-1)\,a$ to $(2p-1)\,a$. Hence the amplitude generated by the pth element of depression at the focus F is

$$S_{pD}=\exp\{ik(ct-\alpha\}\int_{2(p-1)a}^{(2p-1)a}\exp(-ik\beta x)dx$$

$$=\exp\{ik(ct-\alpha)\}\frac{1}{-ik\beta}[\exp\{-ik\beta(2p-1)a\}$$
$$-\exp\{-2ik\beta(p-1)a\}]$$

$$=\frac{1}{-ik\beta}\exp\{ik(ct-\alpha)\}\exp\{-2ik\beta(p-1)a\}\cdot$$
$$\times[\exp(-ik\beta a)-1] \qquad \cdots \qquad (5)$$

The total amplitude generated at the focus (considering upto N depressions) is therefore given by,

$$D=\frac{1}{-ik\beta}\exp\{ik(ct-\alpha+2\beta a)\}\times\{\exp(-ik\beta a)-1\}$$
$$\times[\exp(-2ik\beta a)+\exp(-4ik\beta a)+\exp(-6ik\beta a)+\cdots$$
$$+\exp(-2Nik\beta a)\}.$$
$$=\frac{1}{-ik\beta}\exp\{ik(ct-\alpha)+2\beta a\}\cdot\{\exp(-ik\beta a)-1\}$$
$$\times[\exp(-2ik\beta a)][1+\exp(-2ik\beta a)+\exp(-4ik\beta a)+\cdots$$
$$+\exp\{-2(N-1)ik\beta a\}]$$
$$=\frac{1}{-ik\beta}\exp\{ik(ct-\alpha)\}\{\exp(-ik\beta a)-1\}$$
$$\times\left[\frac{1-\exp(-2Nik\beta a)}{1-\exp(-2ik\beta a)}\right] \quad \cdots \quad (6)$$

Multiplying the complex amplitude by its conjugate and simplifying we get for the real amplitude (D) of vibration of the D-grating at the focus F,

$$D=\frac{a}{\frac{k\beta a}{2}}\cdot\sin\frac{k\beta a}{2}\cdot\frac{\sin Nk\beta a}{\sin k\beta a} \quad \cdots \quad (7)$$

The equation of vibration at the focus F due to D-grating is given by,

$$Y_D=D\cos k(ct-\alpha) \quad \cdots \quad (8)$$

where Y_D is the displacement of the either particle at the focus.

E-type grating : In this type of grating the path difference for the *pth* elevation is given by,

$$\Delta pE=K'G'+H'G'$$
$$=K'G'+L'G'\cos\theta$$
$$=K'G'+(K'G-K'L')\cos\theta.$$
$$=K'G'(1+\cos\theta)-OK'\tan\theta\cos\theta.$$
$$=K'G'(1+\cos\theta)-OK'\sin\theta.$$

Putting $\eta=e(1+\cos\theta)$ and $\beta=-\sin\theta$ we have,

$$\Delta pE=\eta+\beta x \quad \cdots \quad (9)$$

The displacement at the focus F due to an element dx of the incident wave-front at a distance x from 0 is given by,

$$dS_{pE}=\exp\{ik(ct-\eta)\}\exp(-ik\beta x)dx \quad \cdots \quad (10)$$

The total displacement at the focus F due to all elements of the E-grating is to be obtained by integration. The limits of integration for the first, second, third etc. elevations extend from a to $2a$, $3a$ to $4a$, $5a$ to $6a$ etc. The limits of integration of the pth element are $(2p-1)\,a$ to $2pa$.

The displacement due to the pth element of the grating is,

$$S_{pE}=\exp\{ik\,(ct-\eta)\}\int_{(2p-1)a}^{2pa}\exp(-ik\beta x)\,dx$$

$$=\exp\{ik\,(ct-\eta)\}\times\frac{1}{-ik\beta}\times[\exp(-2ik\beta pa)$$
$$-\exp\{-ik\beta\,(2p-1)a\}]$$

$$=\frac{1}{-ik\beta}\exp\{ik\,(ct-\eta)\}\exp(-2ik\beta pa)$$
$$[1-\exp(ik\beta a)] \qquad \cdots \qquad (11)$$

The displacement at the focus F generated by all elevations of the E-grating is obtained by summing over all values of p from 1 to N. The total amplitude S_E so generated is therefore given by,

$$S_E=\frac{1}{-ik\beta}\exp\{ik\,(ct-\eta)\}\{1-\exp(ik\beta a)\}[\exp(-2ik\beta a)$$
$$+\exp(-4ik\beta a)+\exp(-6ik\beta a)+\cdots+\exp$$
$$(-2Nik\beta a)]$$

$$=\frac{1}{-ik\beta}\exp\{ik\,(ct-\eta)\}\times\exp(-2ik\beta a)\times\{1-\exp(ik\beta a)\}$$
$$\times[1+\exp(-2ik\beta a)+\exp(-4ik\beta a)+\cdots+\exp$$
$$\{-2(N-1)\,ik\beta a\}]$$

$$=\frac{1}{-ik\beta}\cdot\exp\{ik\,(ct-\eta-2\beta a)\}\{1-\exp(ik\beta a)\}$$
$$\times\left[\frac{1-\exp(-2Nik\beta a)}{1-\exp(-2ik\beta a)}\right] \qquad \cdots \qquad (12)$$

Multiplying the complex amplitude by its conjugate and simplifying we get for the real amplitude (E) of the E-grating.

$$E=\frac{a}{\frac{k\beta a}{2}}\sin\frac{k\beta a}{2}\cdot\frac{\sin Nk\beta a}{\sin k\beta a} \qquad \cdots \qquad (13)$$

The equation of vibration at the focus F due to the E-grating is given by,

$$Y_E = \frac{a}{\frac{k\beta a}{2}} \cdot \sin \frac{k\beta a}{2} \cdot \frac{\sin Nk\beta a}{\sin k\beta a} \cos (ct - \eta - 2\beta a)$$

$$= E \cos k(ct - \eta - 2\beta a) \qquad \cdots \qquad (14)$$

The resultant intensity expression is obtained by vector addition of Y_D and Y_E. The resultant intensity (J) is the square of the resultant amplitude,

$$\therefore \quad J = (D+E)^2 - 4ED \sin^2 \frac{k}{2}(\delta_1 - \delta_2) \qquad \cdots \qquad (15)$$

where $\delta_1 = -\alpha$ and $\delta_2 = -\eta - 2\beta a$

Substituting the values of δ_1 and δ_2 and of $\alpha = (e+d)(1+\cos\theta)$, $\beta = -\sin\theta$, $\eta = e(1+\cos\theta)$ the expression for intensity becomes

$$J = \left[\frac{4a^2}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2}\right] \times \left[\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)\right] \times \left[\frac{\sin^2\left(\frac{2\pi N a \sin\theta}{\lambda}\right)}{\sin^2\left(\frac{2\pi a \sin\theta}{\lambda}\right)}\right]$$

$$\times \cos^2 \frac{\pi}{\lambda}\{d(1+\cos\theta) + 2a\sin\theta\} \qquad \cdots \qquad (16)$$

The first two factors of the above equation multiplied together represent the diffraction band due to a single slit of width a. The third factor gives the primary and intervening (N-2) secondary spectra. The maximum of the primary spectrum occurs at θ_s given by,

$$2a \sin\theta_s = s\lambda \qquad [s = 1, 2, 3 \text{ etc.}]$$

The first minimum following the s th. central maximum of the primary spectrum occurs at $\Delta\theta_s$ given by,

$$2a \sin(\theta_s + \Delta\theta_s) = \left(S + \frac{1}{N}\right)\lambda \qquad \cdots \qquad (17)$$

so that the half width of the s th. fringe of the primary spectrum is

$$\Delta\theta_s = \frac{\lambda}{2Na\cos\theta_s} \qquad \cdots \qquad (18)$$

The intensity expression given by equation (16) is similar to that of a plane grating multiplied by the $\cos^2$ term. The $\cos^2$

term plotted against has been represented in Fig. 2. For calculations the incident wavelength was taken as 10^{-4} cm, $a=10^{-8}$ cm and $d=\frac{\lambda}{2}=\frac{10^{-4}}{2}$ cm. The data have been presented in the table given below :

$\theta°$	$\text{Cos}^2 x$	$\theta°$	$\text{Cos}^2 x$
0	1·000	26	0·396
2	0·329	28	0·250
4	0·106	30	0·957
6	0·924	32	0·005
8	0·587	34	0·904
10	0·011	36	0·235
12	0·808	33	0·638
14	0·719	40	0·465
16	0·0003	42	0·482
18	0·779	44	0·500
20	0·687	46	0·552
22	0·015	48	0·329
24	0·924	50	0·821

Thus from equation (16) we find that the lamellar grating is equivalent to a combination of plane reflection grating and echelon reflection grating.

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INDIAN JOURNAL OF THEORETICAL PHYSICS
SILVER JUBILEE VOLUME (1953-1977)

An electron fluid model for metals : Application to alkali metals

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[**Abstract** : A simple electron fluid model is proposed for the lattice dynamics of metals. The electronic part of the dynamical matrix satisfies the correct symmetry property of the lattice and the lattice is in equilibrium without recourse to external forces. The contribution of the electron fluid to the elastic constants of metals is carefully developed and it is shown that the elastic constants obtained from the homogeneous deformation theory and the long-wave theory agree when equilibrium condition is used. The model is used to calculate the phonon dispersion of *Li*, *K* and *Rb* and the agreement with the experimental values is quite satisfactory. An interesting fact that emerges out of the present model is that the dominant effect of the free-electrons can be represented by an effective three-body ion-ion interaction which is responsible for the violation of Cauchy symmetry in elastic constants.]

1. *Introduction*

As it is difficult to develop a fundamental microscopic theory for the lattice dynamics of metals, considerable effort has been devoted to the development of phenomenological theories which take into account the effects of conduction electrons on the atomic vibrations. Starting with the works of Fuchs[1] and de Launay[2, 3, 4] many authors developed phenomenological models in which the effect of quasifree electrons on the lattice dynamics was treated as that of a compressible gas[5, 6, 7]. The simplicity of the models is appealing. However all these models

suffer from two major defects. Firstly, the electron gas part of the dynamical matrix does not have the correct symmetry property, and secondly in some of the models the question of equilibrium is inadequately treated[8].

Krebs[9, 10] proposed a model for the lattice dynamics of metals which satisfies the requirement of translational invariance and Chéveau proposed a modification of the Krebs model so that the lattice may be in equilibrium without recourse to external forces. Lax[12] has given a general discussion of free electron metals. All these authors considered the effect of free electrons through the screened coulomb interaction and a central interaction between first and second neighbours. As the screened coulomb interaction is an effective two-body central interaction one cannot expect a breakdown of the Cauchy relation on the basis of such models. One may, of course, obtain an apparent Cauchy violation even with a two-body central interaction if the equilibrium condition is not used or if it is wrongly substituted.

Another defect inherent in the phenomenological models[4, 5, 6] is that, the first order term in the expansion of the energy of the electron fluid is neglected. This will lead to a contribution of K_e (bulk modulus of the electron gas) to C_{11} and C_{12} both in the homogeneous deformation theory and the long-wave theory and no contribution of the electronic part to C_{44}. It can be shown that under these circumstances, elastic contants obtained from the homogeneous deformation theory and the long wave theory will not agree even if equilibrium condition is used. This shows that the first order term in the energy of the electron fluid is important in order to get the correct values of elastic constants.

The model we propose removes all these difficulties. The dynamical matrix satisfies the symmetry property of the lattice. The lattice is in equilibrium without recourse to external forces and the model explains the Cauchy violation. An interesting fact that emerges out of the present model is that the dominant effect of the free electrons can be represented by

an effective three-body ion-ion interaction. In the present paper we develop the theory in the case of b.c.c. metals and calculate the phonon dispersion of *Li*, *K* and *Rb*. To check the expressions for the elastic constants used we have calculated them both from the homogeneous deformation theory and the theory of long-wave and have shown that they agree after substitution of the equilibrium condition.

Pseudopotential calculation upto the second order perturbation terms show that the total energy of the ion plus quasifree electron system may be divided into two parts. One is due to an effective two-body central interaction between the ions and the other consists of several terms all of which depend on the total volume of the lattice, the so called volume dependent terms. We attribute these volume dependent terms to the effect of the electron fluid. Thus the total interaction consists of the two-body central ion-ion interaction which for simplicity is confined to nearest neighbours only in the present calculation and the effective ion-ion interaction that results from the coupling of electron fluid to the motion of the ions.

2. *Energy of the electron fluid*

Treating the quasifree electrons as a fluid of bulk modulus k_e, we expand the energy density of the electrons in powers of volume strain χ and retaining upto the quadratic term we get

$$U^e = U^e_0 - p_e\chi + \tfrac{1}{2}k_e\chi^2 \qquad \cdots \qquad (1)$$

where p_e is the pressure of the electron fluid and U^e_0 is the equilibrium energy density.

The volume strain χ is developed in the electron fluid because the electrons are adiabatically adjusting themselves to the ionic displacement. The coupling between the electron motion and the ionic motion must be such that this adiabatic condition is not violated. The fluid will see the wave motion through the ionic waves and hence it must not be able to distinguish between two waves whose wave vector differ by a reciprocal vector. In

all previous works with electron gas model this crucial point has been ignored and the electron gas is treated as an independent continuum which can distinguish waves of any wavelength. Here we want to ensure that the adiabatic condition is not violated.

We divide the lattice into Wigner-Seitz cells and assume that the average volume strain of the electron fluid in each cell is equal to the local average ionic strain. The average volume strain $<\chi^l>$ of electron in the l-th cell for a cubic crystal can be written as

$$<\chi^l> = \frac{1}{n} \sum_{l',\,\alpha} \frac{u_\alpha(l') - u_\alpha(-l')}{2r_0^2} r_\alpha(ll') \quad \cdots \quad (2)$$

(nearest neighbour l)

where $2r_o$ is the cube edge, n is the number of nearest neighbours l' which contribute non-vanishing terms to the summation in equation (2), for each α and is equal to 8 for b.c.c. crystal, $\underset{\sim}{r}(ll') = \underset{\sim}{r}(l') - \underset{\sim}{r}(l)$ is the equilibrium separation of l and l' ions and $\underset{\sim}{u}(l)$ is the displacement vector of the l-th ion.

Using equation (2) for $<\chi^l>$ we get

$$<\chi^l>^2 = \frac{1}{n^2} \frac{1}{4r_0^4} \sum_{l'l''} \sum_{\alpha\beta} [u_\alpha(l') - u_\alpha(-l')][u_\beta(l'') - u_\beta(-l'')]$$

$$(n.n. \text{ of } l) \qquad r_\alpha(ll') r_\beta(ll'') \quad \cdots \quad (3)$$

$$= \frac{1}{n^2 r_0^4} \sum_{l'l''} \sum_{\alpha\beta} u_\alpha(l') u_\beta(l'') r_\alpha(ll') r_\beta(ll'') \quad \cdots \quad (4)$$

$$(n.n. \text{ of } l)$$

So the quadratic term in the energy of the electron fluid can be written as

$$U^e = \tfrac{1}{2} k_e v_o \sum <\chi^l>^2$$

$$= \tfrac{1}{2} k_e v_o \times \frac{1}{n^2 r_0^4} \sum_{l} \sum_{\substack{l'l'' \\ \alpha\beta}} u_\alpha(l') u_\beta(l'') r_\alpha(ll') r_\beta(ll'')$$

$$(c.n.n. \text{ of } l'l'') \quad \cdots \quad (5)$$

where l is the common nearest neighbour of l' and l'' and v_o is the volume per atom.

3. *Effective three-body ion-ion force constant*

It is seen from equation (5) that the effect of the free electrons can be represented by a three-body ion-ion interaction. The three-body force constant between l' and l'' ions can be written as

$$\Phi^{e}_{\alpha\beta}(l'l'')=\frac{k_e v_0}{n^2 r_0^4}\sum_{l} r_\alpha(ll')r_\beta(ll'') \qquad \cdots \qquad (6)$$

$$(c.n.n. \text{ of } l'l'')$$

For b.c.c. structure $v_0=4r_0^3$ and $n=8$, So we can write

$$\Phi^{e}_{\alpha\beta}(l'l'')=\frac{k_e}{16 r_0}\sum_{l} r_\alpha(ll')r_\beta(ll'') \qquad \cdots \qquad (7)$$

$$(c.n.n. \text{ of } l'l'')$$

For b.c.c. structure non-vanishing force constants exist for atoms of type *a*, *b*, *c* shown in Fig. 1.

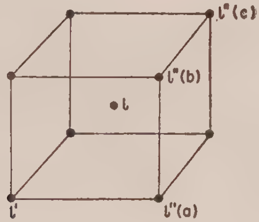

Fig. 1.

The figure shows the positions of the particles for which non-vanishing many-body force constants exist; other particles are obtained from symmetry.

The force constant matrices are given by

$$\Phi^{e}(l'a)=\frac{k_e r_0}{4}\left|\begin{matrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right| \qquad \cdots \qquad (8a)$$

$$\Phi^{e}(l'b)=\frac{k_e r_0}{8}\left|\begin{matrix}-1 & 0 & -1\\ 0 & 1 & 0\\ -1 & 0 & -1\end{matrix}\right| \qquad \cdots \qquad (8b)$$

$$\Phi^{e}(l'c)=-\frac{k_e r_0}{16}\left|\begin{matrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{matrix}\right| \qquad \cdots \qquad (8c)$$

and

$$\Phi^e(l'l') = \frac{k_e r_o}{2} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \cdots \quad (8d)$$

4. *Dynamical matrix*

The contribution to the dynamical matrix because of the force constants given by equations (8a) to (8d) is given by

$$D^e_{\alpha\alpha} = 4k_e r_o \sin^2 q_\alpha r_o \cos^2 q_\beta r_o \cos^2 q_\gamma r_o$$

$$D^e_{\alpha\beta} = k_e r_o \sin 2q_\alpha r_o \sin 2q_\beta r_o \cos^2 q_\gamma r_o \quad \cdots \quad (9)$$

It may be noted that the electronic contributions to the dynamical matrix are invariant with respect to a translation equal to a reciprocal lattice vector which is not the case with other electron gas models.

If we confine the central interaction to nearest neighbours only then the force constant matrix corresponding to central interaction can be expressed in terms of the first and second derivative of the interaction potential $\Phi(r)$. The force constant matrix between nearest neighbours is given by

$$\Phi^c(ll') = \begin{vmatrix} \alpha & \gamma & \gamma \\ \gamma & \alpha & \gamma \\ \gamma & \gamma & \alpha \end{vmatrix} \quad \cdots \quad (10)$$

where
$$\alpha = -\frac{\Phi''}{3} - \frac{2}{3\sqrt{3}} \frac{\Phi'}{r_o}$$

$$\gamma = -\frac{\Phi''}{3} + \frac{1}{3\sqrt{3}} \frac{\Phi'}{r_o} \quad \cdots \quad (11)$$

The dynamical matrix corresponding to the force constant given by equation (10) is

$$D^c_{\alpha\alpha} = -8\alpha(1 - \cos q_\alpha r_o \cos q_\beta r_o \cos q_\gamma r_o)$$

$$D^c_{\alpha\beta} = -8\beta \sin q_\alpha r_o \sin q_\beta r_o \cos q_\gamma r_o \quad \cdots \quad (12)$$

5. *Elastic constants*

From equations (9) and (12) we can obtain the expression for the elastic constants by using the method of long waves and is given by

$$
\begin{aligned}
C_{11} &= \frac{\Phi''}{3r_o} + \frac{2}{3\sqrt{3}}\,\frac{\Phi'}{r_o^{\,2}} + k_e \\
C_{12} &= \frac{\Phi''}{3r_o} - \frac{4}{3\sqrt{3}}\,\frac{\Phi'}{r_o^{\,2}} + k_e \qquad \cdots \qquad (13) \\
C_{44} &= \frac{\Phi''}{3r_o} + \frac{2}{3\sqrt{3}}\,\frac{\Phi'}{r_o^{\,2}}
\end{aligned}
$$

To find the expression for the elastic constants from homogeneous deformation theory we start with the expression for the energy density given by equation (1).

The volume strain is given by

$$
\begin{aligned}
\chi &= \frac{dV}{V} = \mathrm{Det}\,(1+v) - 1 \\
&= (v_{11}+v_{22}+v_{33}) + (v_{11}v_{22}+v_{22}v_{33}+v_{33}v_{11}) \\
&\qquad - (v_{12}^2+v_{13}^2+v_{23}^2) \qquad \cdots \qquad (14)
\end{aligned}
$$

where v_{11}, v_{12} etc. are the infinetesimal strains.

Substituting this value of χ in equation (1) and retaining terms upto quadratic in the infinetesimal strain we get

$$
\begin{aligned}
U^e &= U^e_o - p_e(v_{11}+v_{22}+v_{33}) + \tfrac{1}{2}k_e'(v_{11}^2+v_{22}^2+v_{33}^2) \\
&\quad + p_e(v_{12}^2+v_{13}^2+v_{23}^2) + (k_e-p_e)(v_{11}v_{22}+v_{22}v_{33}+v_{33}v_{11}) \\
&\qquad \cdots \qquad (15)
\end{aligned}
$$

We consider the ionic system to be a cubic solid under pressure p. The energy density of this solid can also be expanded in terms of the infinetesimal strain. Upto quadratic terms in the strain the energy density of the solid is given by

$$
\begin{aligned}
U^s &= U^s_o - p(v_{11}+v_{22}+v_{33}) + \tfrac{1}{2}(C^s_{11}-p)(v_{11}^2+v_{22}^2+v_{33}^2) \\
&\quad + C^s_{12}(v_{11}v_{22}+v_{22}v_{33}+v_{33}v_{11}) + 2\,(C^s_{44}-\tfrac{1}{2}p) \\
&\qquad (v_{12}^2+v_{23}^2+v_{31}^2) \qquad \cdots \qquad (16)
\end{aligned}
$$

where C^s_{11}, C^s_{12} and C^s_{44} are the zero-pressure elastic constants of the solid.

So the energy density of the solid and electron fluid is,

$$U = U^s + U^e = (U^s_o + U^e_o) + \tfrac{1}{2}(C^s_{11} + k_e + p_e)(v^2_{11} + v^2_{22} + v^2_{33})$$

$$+ (C^s_{12} + k_e - p_e)(v_{11}v_{22} + v_{22}v_{33} + v_{33}v_{11})$$

$$+ 2(C^s_{44} + p_e)(v^2_{12} + v^2_{23} + v^2_{31}) \quad \cdots \quad (17)$$

The first order term vanishes because the solid and the electron gas system is under stress free condition, *i.e.* $p + p_e = 0$.

Comparing equation (17) with the expression for the energy density of a solid at zero pressure (obtained by putting $p = 0$ in equation (16)) we get

$$C_{11} = C^s_{11} + k_e + p_e$$

$$C_{12} = C^s_{12} + k_e - p_e$$

$$C_{44} = C^s_{44} + p_e \quad \cdots \quad (18)$$

If the interaction potential be $\Phi(r)$, then the elastic constants as obtained from the method of homogeneous deformation is given by

$$\overset{s}{C}_{\alpha\beta\gamma\delta} = \frac{1}{2v_0} \sum_{l'} D^2 \Phi(r(ll'))\, r_\alpha(ll') r_\beta(ll') r_\gamma(ll') r_\delta(ll') \quad \cdots \quad (19)$$

where

$$D = \frac{1}{r} \frac{d}{dr}$$

For b.c.c. crystal with nearest neighbour interaction we get

$$C^s_{11} = C^s_{12} = C^s_{44} = \frac{\Phi''}{3r_0} - \frac{1}{3\sqrt{3}} \frac{\Phi'}{r_0{}^2} \quad \cdots \quad (20)$$

The equilibrium condition can be written as

$$p_e - \frac{1}{\sqrt{3}} \frac{\Phi'}{r_0{}^2} = 0 \quad \cdots \quad (21)$$

Using equation (18), (20) and (21) we obtain equation (Fig. 13). Thus the elastic constants obtained from the homogeneous deformation theory agree with those obtained from the theory of long waves.

6. *Phonon Dispersion*

Utilising equations (21) and (13) the four parameters of the model Φ'', Φ', k_e and p_e are determined from the experimental values of the harmonic elastic constants and equilibrium lattice separation. The parameters along with the input data are displayed in Table 1. The model parameters

Table 1

Model Parameters and Input Data Used for Their Evaluation.

Metal	Input data: Harmonic elastic constants(18) 10^{12} dyne/cm²			Input data: Lattice(18) constant 10^{-8} cm	Parameters			
	C_{11}	C_{12}	C_{44}	$2r_o$	p_e 10^{12} dyne/cm²	k_e 10^{12} dyne/cm²	Φ' 10^{-4} dyne	Φ'' 10^{4} dyne/cm
K	·0441	·0365	·0310	5·2400	·0038	·0131	·0451	·2237
Li	·1720	·1460	·1280	3·4832	·0130	·0440	·0683	·6235
Rb	·0349	·0294	·0222	5·5718	·0027	·0127	·0362	·1702

are then used to calculate the quasi-harmonic phonon frequencies of *Li*, *K* and *Rb* in the symmetry direction and these are shown

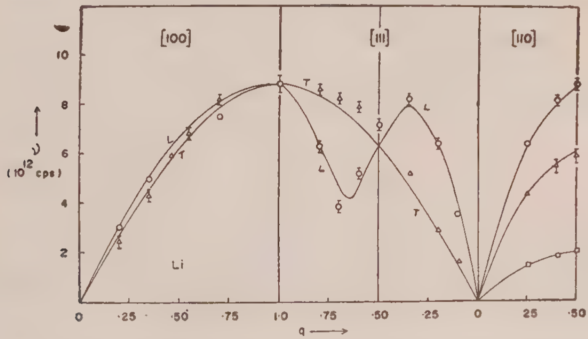

Fig. 2. Phonon dispersion curve of Li in the symmetry directions at 90°K. Solid line represents the calculated curve. Experimental points are taken from Ref. 15.

in Fig. 2, 3, 4 respectively. It is seen that the agreement with experimental values is quite satisfactory. It is to be noted that

the entire phonon spectra in the symmetry directions is predicted though none of the frequencies has been fitted, unlike the majority of the phenomenological models which fit one or more zone boundary frequencies.

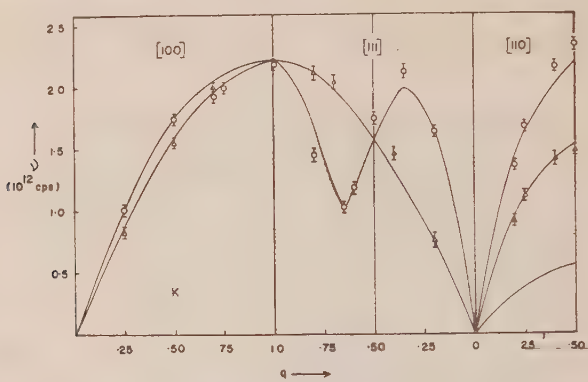

Fig. 3. Phonon dispersion curve of K in the symmetry directions at 9°K. Solid line represents the calculated curve. Experimental points are taken from Ref. 16.

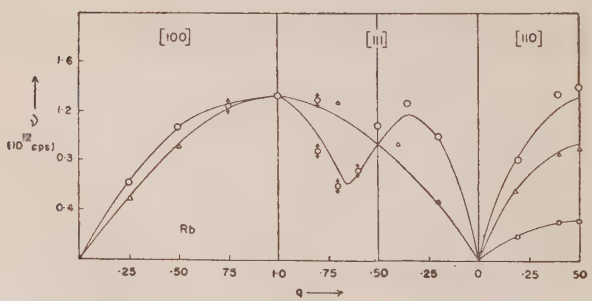

Fig. 4. Phonon dispersion curve of Rb in the symmetry directions at 120°K. Solid line represents the calculated curve. Experimental points are taken from Ref. 17.

Recently I. C. Da Cunha Lima et al[18] have used a five parameter model to calculate the lattice dynamics of *Li*, *K* and *Rb*. Though these authors extended the ion-ion interaction to second neighbours and fitted two of the zone-boundary frequencies, an overall discrepancy of the order of 15·20% still remains.

The present calculation with only four parameters and ion-ion interaction upto first neighbour only gives much better agreement than that obtained by these authors. It is worth mentioning here that a force constant approach requires at least fifth to sixth neighbour ion-ion interaction to explain quantitatively the entire dispersion curves of these metals. In the present case with the inclusion of the effect of electrons, it is sufficient to reduce the interionic interactions to first neighbour only in order to obtain an excellent agreement of both phonon dispersion curves and the elastic constants.

More recently, Dagens et al[14] have calculated the phonon dispersion of *Li* and *K* with a six parameter model pseudo-potential. The parameters of the potential were adjusted to fit the nonlinear self consistent calculation of the charge densities in the metals. From the observed discrepancy in the case of *Li* they concluded that three and four-body forces play a larger role in the lattice dynamics of metals. It is interesting to note that the present model introduces effective ion-ion three-body interaction.

In conclusion we can say that the present model is very simple and at the same time removes all the discrepancies existing in the phenomenological electron gas models. The overall agreement is quite satisfactory. The small remaining discrepancy is likely to be removed by extending the ion-ion interaction to higher neighbours.

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INDIAN JOURNAL OF THEORETICAL PHYSICS
SILVER JUBILEE VOLUME (1953-1977)

Studies on the modulus defect in ferromagnetic materials

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[**Abstract :** In the present paper the author has studied the modulus defect which arises due to the presence of internal strain in a ferromagnetic material. The expression obtained here for modulus defect is new and capable of explaining its variation with magnetisation without going in detail to the domain structure and its complicated orientation theory with magnetisation. The theoretical values of modulus defect in annealed Nickel and in annealed Cobalt obtained in the paper are in beautiful agreement with the experimental values obtained by S. Siegel & S. L. Qiumby[1].]

Introduction

It is well known that ferromagnetic materials exhibit magnetostriction or change in dimension when exposed to a magnetic field. The study of the phenomena pertaining to this effect is especially significant because of the extensive use of ferromagnetic materials in industry. The magnetostriction effect in ferromagnetic material, and the different phenomena relevant to it have been studied by a number of authors including F. D. Smith[2]. S. Butterworth[3], J. M. Ide[4], F. M. Leslie[5], S. K. Ghosh and S. Bera[6].

In the treatment of the problem we have considered here that a magnetic field is applied along the axis of a rod made of magnetostrictive material. The presence of domains within the rod complicates the problem concerning the vibration over the case of a non-magnetic rod. In this context we may reasonably assume that the domains are evenly distributed in a ferromagnetic material and the walls separating them are also distributed in the same manner. In the unmagnetised

state, due to the passage of an acoustical wave through the material, all the domain walls will vibrate about their equilibrium positions along with the medium of the material—in the typically acoustic manner. Presence of any external steady magnetic field will cause some of the domains to orient themselves along the favourable direction while the passage of an acoustical wave through the material will force to vibrate the non-oriented walls as before. The availability of the number of non-oriented walls will depend on the strength and direction of the external field. Thus the presence of an external magnetic field changes the internal strain of the material and consequently affects its modulus.

It is known that magnetostriction in the domains causes a change in dimension of the unit cell in the direction of polarisation and an opposing change in direction perpendicular to the polarisation. W. P. Mason[7] has explained this effect as an even powered function so that the effect of applied stress is the same on each of the oppositely directed domains on the two sides of the wall. The experimental observation of the average macroscopic effect of holomagnetisation in a ferromagnetic rod placed axially in a magnetic field being independent of the sense of the field producing either increase or decrease in length of the rod depending on the material. K. C. Kar[8] has extended this idea to the case of a polycrystalline ferromagnetic material in which the internal strain produced due to the orientation of the domains in presence of an external magnetic field has been expressed as an even powered function of the flux density (B). In low biasing fields, far below its saturation value, the internal strain produced has been expressed here as a function of the square of the flux density.

Analysis of the problem

For mathematical convenience, it is a common practice in connection with the problem on vibration to consider the relation between stress (σ) and the strain (ϵ), in a solid to be of the form :

$$\sigma = Y\epsilon \quad \cdots \quad (1)$$

where Y is the Youngs modulus of the material.

In the present problem the strain produced in the rod made of ferromagnetic material consists of two parts, *viz.* the strain that due to the mechanical stressing (ϵ_{mech}) and that due to the change in dimension due to magnetostriction under a steady biasing magnetic field (ϵ_{mag}).

So equation (1) may be written as

$$\sigma = Y(\epsilon_{mag} + \epsilon_{mech}) \quad \cdots \quad (2)$$

It is an experimental fact that the average macroscopic effect of the holomagnetisation is independent of the direction of the applied field and may either increase or decrease the original length of the rod depending on the nature of the material. Hence the magnitude of the strain produced due to the characteristics of domains should in general, be expressed as an even powered function of the flux density.

For the values of B much less than its saturation value we consider this strain to be proportional to B^2 and hence.

$$\epsilon_{mag} = \chi B^2 \quad \cdots \quad (3)$$

Where χ is the proportionality constant. Thus the stress-strain relation (2) becomes

$$\sigma = Y(\epsilon_{mech} + \chi B^2)$$

$$= Y\frac{d\omega}{ds} + \chi Y B^2 \quad \cdots \quad (4)$$

where ω is the longitudinal particle displacement and s is the longitudinal co-ordinate along the rod.

The equation of longitudinal motion in a thin tube or rod is given by

$$\rho\frac{d^2\omega}{dt^2} = \frac{d\sigma}{ds} \quad \cdots \quad (5)$$

Substituting the value of σ from equation (4) the equation (5) becomes

$$\rho\frac{d^2\omega}{dt^2} = Y\left(\frac{d^2\omega}{ds^2}\right) + \lambda_m\frac{dB}{ds} \quad \cdots \quad (6)$$

where λ_m is the magnetostriction constant and is given by

$$\lambda_m \doteq 2\chi BY \quad \cdots \quad (7)$$

Here B is taken as the mean flux density over the distance ds.

The magnetostriction effect being reversible the total change

in flux density may be taken to be the sum of that due to the change in biasing field and that due to the mechanical stressing. As the variation in biasing field (δH) over the distance δs may be taken constant, the change in flux density (δB) is only due to the mechanical stressing. It is known (S. Butterworth and F. D. Smith[8] that the strain at any point produces a localised field which is given by $H_\Delta = K\ \epsilon_{mech}$. In electromagnetic unit it is seen that $K = 4\pi\lambda_m$ and hence

$$H_\Delta = 4\pi\lambda_m \epsilon_{mech}$$

This will consequently produce a change in flux density B_Δ which is given by

$$B_\Delta = 4\pi\lambda_m \mu_\Delta \epsilon_{mech} \qquad \cdots \qquad (8)$$

where μ_Δ is the incremental permeability at the point B of the B- H curve of the specimen. If we apply low amplitude mechanical stressing the field produced is very small and μ_Δ approaches a limiting value μ_r, the reversible permeability of the specimen.

$$\text{So,} \quad \frac{dB}{ds} = 4\pi\lambda_m \mu_r \frac{d^2\omega}{ds^2} \qquad \cdots \qquad (9)$$

and equation (6) becomes

$$\rho \frac{d^2\omega}{dt^2} = Y_1 \frac{d^2\omega}{ds^2} \qquad \cdots \qquad (10)$$

where
$$Y_1 = Y + 4\pi\mu_r\lambda_m^2 \; ; \qquad \cdots \qquad (11)$$

With the help of equations (3) and (7) the equation (11) may be rewritting as

$$\frac{\Delta Y}{Y} = 16\pi\mu_r Y \frac{\epsilon^2_{mag}}{B^2} \qquad \cdots \qquad (12)$$

where $\Delta Y = Y_1 - Y$ is the change in modulus of the material under external magnetic field.

Discussions

Equation (12) represents the modulus defect as a function of magnetisation of a ferromagnetic material. The magnitude of this defect may be calculated if the values of ϵ_{mag}, B and Y of the material be known. We have taken those values from

different literature R. M. Bozorth[9], A.B. Wood[10], R. Street[11] and are given in Table I & II for Nickel and Cobalt.

Table I

Nickel : Nickel being highly permeable material we consider approximately, $\frac{B}{B_s} \simeq \frac{I}{I_s}$;

$Y = 20.2 \times 10^{11}$ dynes/cm^2 ;

H in Oersted	0·5	1	1·5	2	2·05	3	
B in Gauss $\times 10^{-3}$	0·75	1·97	2·8	3·1	3·6	3·8	$B_s = 5$
μ_r	245	160	115	80	70	60	
$\frac{B}{B_s} \simeq \frac{I}{I_s}$	0·125	0·394	0·56	0·62	0·72	0·76	
$\epsilon_{mag} \times 10^6$	0·4	2·2	4·5	7	9·2	11·5	

Table II

Cobalt :

$Y = 21 \times 10^{11}$ dynes/cm^2

H in Oersted	20	40	50	100	150	200	250	
B in Gauss $\times 10^{-3}$	4·5	6·2	7	8·65	8·7	10·7	11·5	
I in C.G.S. Unit	35	100	170	240	325	380	430	$I_s = 1422$
I/I_s	0·025	0·07	0·12	0·17	0·23	0·27	0·30	
$\epsilon_{mag} \times 10^6$	0·15	0·4	0·6	1	1·6	2·2	2·9	
μ_r	83	81	75	68	58	52	45	

Figs (1) and (2) show the variation of modulus defect with magnetisation in annealed Nickel and annealed Cobalt along with the experimental values of S. Siegel & S. L. Quimby[1]. The figures show good agreement between the experimental values and the values calculated from present theory.

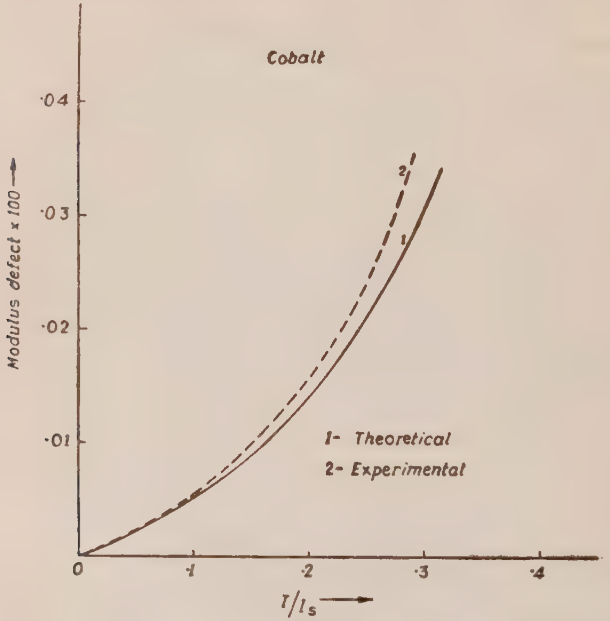

Fig. .2

The observed deviation of the calculated values of modulus defect from the experimental data may be attributed to the fact that the relevant data have been taken from different literature, all of which do not correspond to same experimental condition (like temperature, purity, grain size, texture and crystal perfections of the material).

Again the present theory has been deduced for low biasing so that at higher magnetisation it can not be expected to follow the experimental results accurately. Due to the insufficient data in

hand it is not possible at present to calculate the modulus defect for all the common ferromagnetic materials.

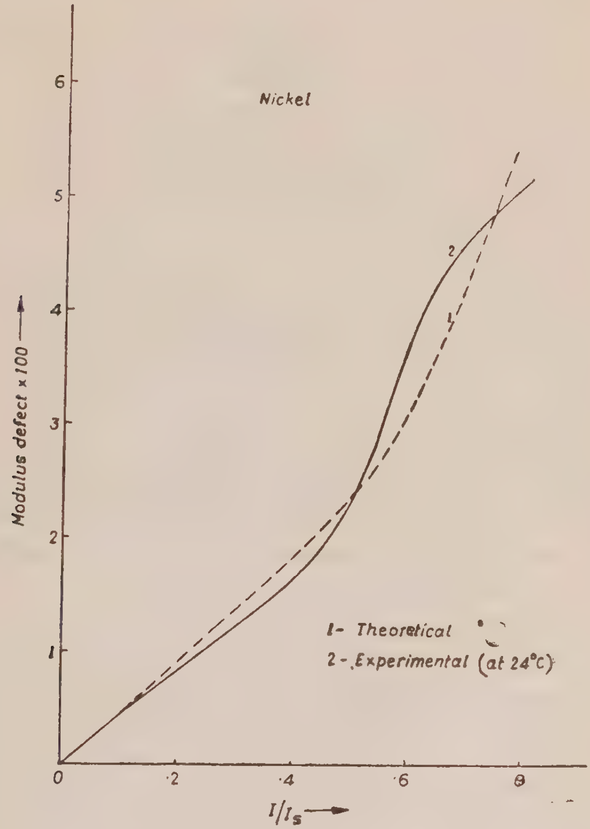

Fig. 1

In the present paper the phenomenon of variation of modulus defect with the frequency of stressing has not been considered. This problem has been tackled by the author from the stand point of pinning of domain walls by dislocations present in the crystal of ferromagnetic material and it will be communicated soon.

The author takes this opportunity to thank Dr. S. K. Ghosh, Reader of Physics, Jadavpur University for his kind suggestions and helpful discussions.

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INDIAN JOURNAL OF THEORETICAL PHYSICS
SILVER JUBILEE VOLUME (1953-1977)

Amplification of longitudinal acoustic waves in thermopiezosemiconductors

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[**Abstract :** This paper is an attempt to study amplification of longitudinal acoustic waves in a thermopiezosemiconducting body. The theoretical analysis, as undertaken in this problem, makes use of the basic equations of mechanical, electrical and thermal-continua, supplemented by appropriate constitutive relations, This investigation for amplification of acoustic waves leads to two critical drift velocities. The estimated lower value is of the order of sound velocity and the upper value is a quantity much more greater than the sound velocity. The amplification is shown to be brought about when the drift velocities lies within these two critical values.]

1. *Introduction*

An important problem of acoustics is the propagation and in particular, amplification or attenuation of waves in a variety of media. One of the means of amplifying acoustic waves is obtained in a piezoelectric semiconductor by a d.c. electric field, *vide* Hutson, McFee and White[1]. The attempt to study amplification of acoustic waves by applying d.c. field in a non-piezoelectric semiconductor by Weinreich[2] has led Hutson and White[3] to study propagation of acoustic waves in piezoelectric semiconductors without being acted upon by a d.c. field. It is well-known that the direct current flowing through a piezoelectric semiconductor in the presence of an acoustic wave gives rise to a travelling a.c. field which, in turn, interacts with acoustic waves. When the drift velocity of the electron exceeds the velocity of sound, amplification ensues.

Of all investigations regarding the amplification of waves, that of longitudinal acoustic waves in piezoelectric semiconductors is important because of their applications as tools in ultrasonics and as devices, such as delay lines and amplifiers, *vide*, White[4]. One has to take into account in these investigations the coupling between the electron and the lattice arising out of the piezoelectric effects. As far as the present authors are aware, none of these attempts excepting the one by Sinha and Gupta[5]. have reckoned thermal considerations affecting the constitutive character of the piezosemiconductors. The present paper is an attempt of this sort and seeks to investigate amplification of longitudinal acoustic waves in a thermopiezosemiconductor, *vide*, by Sinha and Gupta[5]. The analysis proceeds on the lines shown by White[4], White and Hutson[8] and Seeger[6]. The basic equations in the absence of the thermal field are set out in Seeger[6]. The constitutive relations for the thermopiezoelectric material are taken from Mindlin[10] and the Fourier law of heat conduction is modified in order to take into account Peltier effect. As in White[4] and Seeger[6], unidirectional waves have been investigated and it is assumed that the frequency of electron collision is very large compared to the frequency of wave motion.

In course of the findings of this paper, it has been shown that under some approximations there exist two critical drift velocities for amplification of acoustic waves. The estimated lower value is of the order of sound velocity and the upper value is the product of sound velocity and a quantity much greater than unity. The amplification of waves is possible when the drift velocity lies within these two values. The analysis reveals a feature that distinguishes a piezosemiconductor from a thermopiezosemiconductor to the extent it brings out limits of critical velocities for amplification.

2. *Statement of the problem and basic equations*

Let us consider a thermopiezosemiconductor. Let a longitudinal acoustic wave be propagated through such a medium. The medium being thermopiezoelectric, the sound wave is

accompanied by electrokinetic-and thermal-waves and hence, the coupling between the three. The medium is subjected to a d.c. electric field which is one of means of amplifying acoustic waves, *vide*, Hutson, McFee and White[1]. Our problem is to investigate amplification of longitudinal waves which propagate through thermopiezosemiconductors.

The basic equations describing the interaction of mechanical-thermal-and electrical-fields in the medium are those formed by equations of electricity, of heat, of mechanical motion supplemented by constitutive relations. In the present problem, it is assumed that the electron mean free path is short compared with the wavelength of sound, as in an ultrasonic amplifier, *vide*, White[4]. We choose the direction of propagation of longitudinal waves as the direction of x-axis.

The constitutive relations of the material, connecting the mechanical stress T, the electric displacement D, the entropy density σ, the electrical intensity E, the strain S and the perturbed temperature θ, are given by, *vide*, Mindlin[10].

$$T=c_1S-e_{pz}E-\lambda\theta, \qquad \ldots \qquad (1)$$

$$D=\epsilon E+e_{pz}S+p\theta. \qquad \ldots \qquad (2)$$

$$\text{and } \sigma=\lambda s+pE+a\theta \qquad \ldots \qquad (3)$$

where c_1 is the elastic constant, ϵ the dielectric permittivity, a the thermal constant, e_{pz} the piezoelectric constant, λ the thermo-elastic constant and p the thermo-electric constant.

The Fourier law of heat conduction, *vide*, Seeger[7], is given by

$$\frac{\partial q_1}{\partial t}=-k\frac{\partial\theta}{\partial x}+\pi J, \qquad \ldots \qquad (4)$$

where q_1 is the heat flux and k and π are heat conduction co-efficient and Peltier co-efficient.

In the equation (4), the first term of right hand side represents the total heat conduction by phonous and by electrons when there is no electric current and the second term represents the heat conduction by electric current.

The relation between the entropy density σ and the heat flux q_1, is given by, *vide*, Mindlin[10]

$$\frac{\partial^2 q_1}{\partial x\partial t}=-\Theta\frac{\partial\sigma}{\partial t}, \qquad \ldots \qquad (5)$$

where $\Theta=\Theta_0+\theta$, Θ_0 being the reference temperature. The equation of mechanical motion is given by

$$\frac{\partial T}{\partial x}=\rho\frac{\partial^2(\delta x)}{\partial t^2}, \qquad \cdots \quad (6)$$

where ρ is the material density, δx the displacement (mechanical). Let us denote the mean carrier density by n_0 and the instantaneous local density by n_0+fn_s, en_s being the space charge and f, the fraction of the space charge which contributes to conduction process. The Gauss equation and the equation of continuity of charge flow for unidirectional motion (in the x-direction) are given by, *vide*, Seeger[6]

$$\frac{\partial D}{\partial x}=en_s, \qquad \cdots \quad (7)$$

$$e\frac{\partial n_s}{\partial t}+\frac{\partial J}{\partial x}=0, \qquad \cdots \quad (8)$$

where $e<0$ is taken for electrons. The current density for an n-type semiconductor in the absence of thermal attributes is well known, *vide*, Seeger[6]. The modified form is taken as

$$J=|e|(n_0+fn_s)\mu(\Theta)E-eD_n(\Theta)\frac{\partial(n_0+fn_s)}{\partial x}$$
$$+eD_s(\Theta)(n_0+fn_s)\frac{\partial\theta}{\partial x}, \qquad \cdots \quad (9)$$

where the second and the third terms are due to diffusions caused by concentration gradient and thermal gradient respectively.

The Peltier co-efficient, as introduced in the equation (4), is given by, *vide*, Seeger[7]

$$\pi=S_n\Theta, \qquad \cdots \quad (10)$$

where S_n is the thermo-electric power of the material. For a non-degenerate n-type semiconductor the thermoelectric power is given by, *vide*, Seeger[8]

$$S_n=-(K_B/|e|)(r+5/2-\zeta_n/K_B\Theta), \qquad \cdots \quad (11)$$

where ζ_n denotes Fermi energy relative to conduction band and for a non-degenerate semiconductor $\zeta_n<0$ and $|\zeta_n|>>K_B\Theta$, so that the value of thermoelectric power S_n is negative in the present situation ; r satisfying the inequality $-\frac{1}{2}\leqslant r\leqslant\frac{3}{2}$, *vide*, Seeger[9].

Let us now reduce the above equations in some suitable form that would facilitate our solution. Differentiating the equation (4) with respect to x and using the equations (5) and (11), we get

$$\Theta\frac{\partial\sigma}{\partial t}=K\frac{\partial^2\theta}{\partial x^2}-\frac{\partial}{\partial x}(S_n\Theta J)$$

$$\text{or,}\quad \Theta_o\frac{\partial\sigma}{\partial t}=K\frac{\partial^2\theta}{\partial x^2}-S_n\Theta_o\frac{\partial J}{\partial x}-S_nJ_o\frac{\partial\theta}{\partial x},\quad\cdots\tag{12}$$

where $\quad J_o=|e|\,n_o\mu(\Theta_o)E_o.$

Again differentiating the equation (7) with respect to t and using the equations (8) and (9), we get

$$\frac{\partial^2 D}{\partial x\partial t}=-\frac{\partial}{\partial x}\left[\left\{\sigma'(\Theta)+(e/|e|)\mu(\Theta)\frac{\partial D}{\partial x}\right\}E\right]+D_n(\Theta)f\frac{\partial^3 D}{\partial x^3}+$$
$$+f\frac{\partial^2 D}{\partial x^2}\frac{\partial D_n(\Theta)}{dx}-e(n_o+fn_s)D_s(\Theta)\frac{\partial^2\theta}{\partial x^2}-fD_s(\Theta)\frac{\partial^2 D}{\partial x^2}\frac{\partial\theta}{\partial x}$$
$$-e(n_o+fn_s)\frac{\partial\theta}{\partial x}\frac{\partial D_s(\Theta)}{\partial x},\quad\cdots\tag{13}$$

where $\quad \sigma'(\Theta)=|e|\,n_o\mu(\Theta).$

For an acoustic wave in the form given by

$$\delta x=\delta x_1e^{i(\omega t-qx)},\quad\cdots\tag{14}$$

we take

$$E=E_o+E_1e^{i(\omega t-qx)},$$
$$\theta=\theta_1e^{i(\omega t-q\omega)},\quad\cdots\tag{15}$$

where the amplitudes of the perturbed parts are small.

Similarly, we take,

$$D=D_o+D_1e^{i(\omega t-qx)},$$

which becomes

$$D=D_o+(\epsilon E_1-iqe_{pz}\delta x_1+p\theta_1)e^{i(\omega t-qx)},$$

where the equation (2) is considered.

Thus from the equation (13), we obtain

$$E_1\left\{\omega-\frac{e}{|e|}\mu fqE_o-iD_nfq^2-i\sigma'\right\}-iqe_{pz}\left(\omega-\frac{e}{|e|}\mu fqE_o\right.$$
$$\left.-iD_nfq^2\right)\delta x_1+\theta\left\{p\left(\omega-\frac{e}{|e|}\mu fqE_o-iD_nfq^2\right)-qen_oD_s\right.$$
$$\left.+\frac{il\sigma'E_o}{\Theta_o}\right\}=0,$$

which we can write as

$$(\epsilon R-i\sigma')E_1-iqe_{pz}R\delta x_1+(pR+N)\theta_1=0, \quad \dots \quad (16)$$

where

$$R=\omega-\frac{e}{|e|}\mu fqE_o-iD_nfq^2,$$

$$N=\frac{il\sigma' E_o}{\Theta_o}-qen_oD_s.$$

In the equation (16), we have neglected on account of smallness the products of E_1, δx_1 and θ_1 and $\mu(\Theta)=\mu\left(1-l\frac{\theta}{\Theta_o}\right)$, assuming $\mu(\Theta)$ to be proportional to Θ^{-l}, $1<l<2$, *vide*, Kireev[11] and also $\sigma'=|e|n_o\mu$, μ being the value of mobility constant at $\Theta=\Theta_o$.

Again using (14) and (15) in the equation (12), we get

$$\Theta_o(\lambda\omega q\delta x_1+i\omega pE_1+ia\omega)=-kq^2\theta_1+S_n\Theta_o\Big[\frac{e}{|e|}\mu fq^2(\epsilon E_1$$

$$-iqe_{pz}\delta x_1+p\theta_1)E_o-i\sigma' qE_1-\frac{il\sigma' E_oq}{\Theta_o}\theta_1+iD_nfq^3$$

$$(\epsilon E_1-iqe_{pz}\delta x_1+p\theta_1)+q^2en_oD_s\theta_1\Big]+i\beta|e|n_o\mu E_oq\theta_1=0,$$

$$\text{Or,}\quad X\delta x_1+YE_1+Z\theta_1=0 \quad \dots \quad (17)$$

where

$$X=\lambda\omega q+iS_n\frac{e}{|e|}\mu fq^3e_{pz}E_o-S_nD_nfq^4e_{pz},$$

$$Y=i\omega p-S_n\frac{e}{|e|}\mu fq^2\epsilon E_o-i\sigma' pS_n-iD_nfq^3\epsilon S_n,$$

and

$$Z=\frac{kq^2}{\Theta_o}+ia\omega-S_n\frac{e}{|e|}\mu fq^2pE_o+\frac{i(l-1)S_n\sigma' E_oq}{\Theta_o}$$

$$-iD_nfq^3pS_n-q^2en_oD_sS_n.$$

Similarly, from the equations (1) and (6), we obtain

$$(\omega^2-u_s^2q^2)\delta x_1+\frac{iqe_{pz}}{\rho}E_1+\frac{iq\lambda}{\rho}\theta_1=0, \quad \dots \quad (18)$$

where $u_s=\sqrt{\dfrac{c_1}{\rho}}$.

The equations (16), (17) and (18) constitute the basic equations in reduced forms.

3. *Solution*

From the equations (16), (17) and (18), we get the following dispersion equation

$$(\omega^2-u_s{}^2q^2)[Y(pR+N)-Z(\epsilon R-i\sigma')]$$
$$+\frac{iqe_{pz}}{\rho}[-iqe_{pz}RZ-X(pR+N)]+\frac{iq\lambda}{\rho}[X(\epsilon R-i\sigma')$$
$$+iqe_{pz}RY]=0, \quad \cdots \quad (19)$$

Let us now consider the solution of the dispersion equation (19) by perturbation method, as in Kaliski[12].

We set

$$\omega(q)=\omega_s(q)+\eta(q),\ \omega_s(q)=qu_s, \quad \cdots \quad (20)$$

and

$$\eta=\eta_1+i\eta_2, \quad \cdots \quad (21)$$

η_1 and η_2 being real quantities.

We then get from the equation (19), after simplification, the equation

$$2u_s\rho\eta(A+iB)=C+iD, \quad \cdots \quad (22)$$

where

$$A=v_0\left[\frac{\rho_0}{\Theta_0}\left\{l\omega_s p+\sigma' qS_n+(2l-1)fq^3\epsilon S_n\right\}+\epsilon f\ \frac{Kq^3}{\Theta_0}\right]$$
$$-\omega_s\left[\frac{\epsilon Kq^2}{\Theta_0}+a(\sigma'+\epsilon D_nfq^2)-\epsilon q^2\rho_0 D_sS_n\right],$$

$$B=\frac{S_nfq^2\epsilon\rho_0v_0{}^2}{\Theta_0}+v_0\left[-2\sigma' q^2S_npf+a\omega_s\epsilon fq-\frac{\epsilon(1-l)S_n\rho_0q\omega_s}{\Theta_0}\right]$$
$$+2\sigma' q^2S_n\rho_0D_s+\sigma' qS_np\omega_s-a\omega_s{}^2\epsilon+\frac{Kq^2}{\Theta_0}(\sigma'+\epsilon D_nfq^2),$$

$$C=\left(\frac{Kq^4}{\Theta_0}fe_{pz}^2+\frac{l\lambda e_{pz}q\omega_s\rho_0}{\Theta_0}\right)v_0-\left(\frac{Kq^3}{\Theta_0}\ e_{pz}^2\omega_s+\lambda^2q\omega_s\sigma'\right),$$

and,

$$D=-\frac{q^3fe_{pz}^2S_n\rho_0(2l-1)v_0^2}{\Theta_0}+\left[\omega_sq^2f\left(e_{pz}^2a+\lambda^2\epsilon\right)\right.$$
$$\left.+\frac{(l-1)q^2e_{pz}^2S_n\rho_0\omega_s}{\Theta_0}\right]v_0-q^2\lambda e_{pz}\omega_s\sigma' S_n$$
$$-w_s^2q\left(e_{pz}^2a+\lambda^2\epsilon\right) \quad \cdots \quad (23)$$

where $\rho_o = |e| n_o$ and $v_o = -\mu E_o$, v_o being the drift velocity of the carrier. In arriving at the equation (22), we have neglected third order products of e_{pz}, p, λ, D_n, D_s and also the second order products of e_{pz}, p etc. with η.

From the equation (22), we get the amplification co-efficient η_2 as

$$\eta_2 = \frac{AD - BC}{2u_s\rho(A^2 + B^2)} \quad \cdots \quad (24)$$

In (24), A and C are linear in v_o whereas B and D, quadratic in v_o, so that the numerator of the right hand side of (24) is cubic in v_o. So we can write

$$\eta_2 = \frac{a_o v_o^3 + 3b_o v_o^2 + 3c_o v_o + d_o}{2u_s\rho(A^2 + B^2)} \quad \cdots \quad (25)$$

The cubic in the numerator is too unwieldy (on account of involvement of parameters) to determination of critical drift velocity. Yet, we can estimate its idea under some approximations. We can do so by taking q to be small. Considering then the appropriate dominant terms, we find that

$$a_c = \frac{\beta \omega_s^3 q \epsilon \rho_o^2 \lambda e_{pz} l}{\Theta_o^2},$$

$$b_o = -\frac{\omega_s^2 q^2 a' \epsilon \lambda e_{pz} \rho_o l}{3\Theta_o},$$

$$c_o = \frac{\omega_s^3 q a \lambda e_{pz} \epsilon l \rho_o}{3\Theta_o},$$

$$\text{and,} \quad d_o = a^2 \sigma' \omega_s^3 q e_{pz}^2,$$

where we set $S_n = -\beta (\beta > 0)$ and $a' = a\left\{1 - \frac{(l-1)\beta}{\Theta_o a}\right\}$, assuming that $\frac{\rho_o}{\Theta_o} << 1$ and $f \approx 1$.

We can get an estimate of critical velocity if we analyse the roots by solving the cubic equation,

$$a_o v_o^3 + 3b_o v_o^2 + 3c_o v_o + d_o = 0$$

by Cardan's method. This equation reduces to the form

$$v'^3_o + 3Hv'_o + G = 0$$

where $v'_o = a_o v_o + b_o$

and $$H = -\frac{\omega_s^4 q^4 \rho_o^2 \epsilon^2 \lambda^2 e_{pz}^2 l^2}{\Theta_o^2}\left(\frac{a'^2}{9} - \frac{\beta\rho_o a}{3\Theta_o}\right),$$

$$G = -\frac{\omega_s^6 q^6 \epsilon^3 \rho_o^3 \lambda^3 e_{pz}^3 l^3}{\Theta_o^3}\left(\frac{2a'^3}{27} - \frac{aa'\beta\rho_o}{3\Theta_o}\right).$$

If β is of the same order of a, then $a' \approx a$

and $$G^2 + 4H^3 \approx -\frac{1}{27}\frac{a^4\beta^2\rho_o^2}{\Theta_o^2}\cdot\frac{\omega_s^{12} q^{12} \epsilon^6 \rho_o^6 \lambda^6 e_{pz}^6 l^6}{\Theta_o^6}$$

= a negative quantity

Hence all the roots of the cubic equation $a_o v_o^3 + 3b_o v_o^2 + 3c_o v_o + d_o = 0$, are real. Since the equation has only two changes of signs, it has two positive roots. Also we put

$$K = \sqrt{-(G^2+4H^3)} \approx \frac{1}{3\sqrt{3}}\frac{a^2\beta\rho_o}{\Theta_o}\cdot\frac{\omega_s^6 q^6 \epsilon^3 \rho_o^3 \lambda^3 e_{pz}^3 l^3}{\Theta_o^3},$$

and $$\tan\phi = -\frac{K}{G} \approx \frac{3\sqrt{3}\beta\rho_o}{2a\Theta_o},$$

so that ϕ can be taken to be small.

Hence the two positive roots of the cubic equation are given by

$$v_{o_{cr_1}}=\frac{1}{a_o}\left[-b_o+2(-H)^{\frac{1}{2}}\cos\frac{\phi}{3}\right]$$

$$\approx\left(\frac{1}{3}+\frac{2}{3}\cos\frac{\phi}{3}\right)\frac{a\Theta_o}{\beta\rho_o}u_s=Mu_s$$

and $$v_{o_{cr_2}}=\frac{1}{a_o}\left[-b_o-2(-H)^{\frac{1}{2}}\cos\frac{\pi+\phi}{3}\right]$$

$$\approx\left[\frac{1}{3}-\frac{2}{3}\cos\left(\frac{\pi}{3}+\frac{\phi}{3}\right)\right]\frac{a\Theta_o}{\beta\rho_o}u_s=Nu_s,$$

where $$M=\left(\frac{1}{3}+\frac{2}{3}\cos\frac{\phi}{3}\right)\frac{a\Theta_o}{\beta\rho_o},$$

and, $$N=\left[\frac{1}{3}-\frac{2}{3}\cos\left(\frac{\pi}{3}+\frac{\phi}{3}\right)\right]\frac{a\Theta_o}{\beta\rho_o}.$$

Evidently, $M>>1$ and also $M>>N$.

As ϕ is small, so that $\sin\phi=\phi$, $\tan\phi=\phi$ and $\cos\phi=1$ we get,

$$v_{o_{cr1}}\approx Mu_s\approx\frac{a\Theta_o}{\beta\rho_o}u_s=M'u_s(M'>>1)$$

and, $$v_{o_{cr2}}\approx Nu_s\approx\frac{3}{2}u_s\backsim u_s.$$

Since the positive roots of the cubic equation are the critical drift velocities, we can say that there exist two critical drift velocities $v_{o_{cr1}}$ and $v_{o_{cr2}}$ for amplification of acoustic waves in this case and amplification is possible if the drift velocity lies within these two values ($\because$ $a_o>0$) ; otherwise, there is attenuation of waves. Though it is not possible to get exact values of the critical velocities, we see that the lower critical value is of the order of second velocity and the upper value is much more greater than sound velocity. Hence, in the present situation, amplification of waves begins when drift velocity exceeds a value of the order of sound velocity and it continues until the drift velocity reaches a value $M'u_s(M'>>1)$ more greater than sound velocity ; and for much higher values ($>M'u_s$) attenuation of waves occurs.

Conclusions

Summing up, the analysis shows that the amplification coefficient of longitudinal acoustic waves in a thermopiezosemiconducting body depends on a cubic expression of drift velocity, as borne out by the numerator of the equation (25) which is obviously dominant solely for the determination of critical drift velocity, Under the approximation of smallness of wave number and some other approximations, we see that there exist only two critical drift velocities for amplification of acoustic waves. The estimated lower value is of the order of sound velocity and the upper value is the product of sound velocity and a quantity much more greater than unity. The amplification begin when the drift velocity exceeds the lower value and continues until the drift velocity reaches the upper value. For all other values of drift velocity, which lie outside this domain, there is always attenuation of waves. The distinguishing feature between a piezosemiconductor and a thermopiezosemiconductor under present assumptions is that, unlike the latter case, there exists no upper limit of critical value for amplification in the former case, although the lower limit in both the cases are of the same order.

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INDIAN JOURNAL OF THEORETICAL PHYSICS
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On the application of Struve function to a physical problem

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[**Abstract :** The dependence of magnetic anisotropy of carnauba wax magneto-electret on forming field strengths has been investigated. The Curve exhibits an oscillatory nature with a number of peaks at specified field strengths. The Struve function $H_n(x)$ has been computed to simulate the experimental curve. It is found that for $n=1{\cdot}5$, the agreement is satisfactory.]

In general, if a liquefied substance containing magnetically anisotropic molecules is subjected to the influence of a superimposed magnetic field, an orientation effect is most likely to take place. Such orientation, may under suitable circumstances, produce induced magnetic anisotropy in the solidified state of the material. We have made a systematic study of the dependence of magnetic anisotropy of carnauba wax magneto-electret prepared under different forming magnetic fields ranging from 3·6 to 16·6 Kilo-Oersteds. The solid curve of Fig. 1 which embodies the results of our measurements shows that during the initial stage the magnetic anisotropy increases with the rise of the forming field but subsequently reveals an oscillatory

nature showing peaks at specific field intensities somewhat resembling the oscillating magnetic susceptibility of Bismuth at low tempertures in the De Haas-Van Alphen effect[1]. The nature of the experimental curve suggests the existence of a quantum mechanical effect inherent in the phenomenon.

We now proceed to apply the concept of Struve function to obtain a curve which can best be fitted to the experimental curve. It is known that the integral representation of the Struve function[2] is

$$H_n(z)=\frac{z(z/2)^n}{\sqrt{\pi}\Gamma(n+1/2)}\int_0^1 (1-t^2)^{n-1/2}\sin(zt)\,dt, \quad \cdots \quad (1)$$

with the condition $n>-1/2$. The function comes out of the general solution of the equation

$$z^2\frac{d^2w}{dz^2}+z\frac{dw}{dz}+(z^2-n^2)w=\frac{4(z/2)^{n+1}}{\sqrt{\pi}\Gamma(n+1/2)}. \quad \cdots \quad (2)$$

The solution of Eq. (2) is

$$w=a\,J_n(z)+b\,Y_n(z)+H_n(z), \quad \cdots \quad (3)$$

where a and b are constants, $J_n(z)$ and $Y_n(z)$ are the Bessel functions of the first and second kinds respectively, while $H_n(z)$ is the Struve function. It is important to note that the nature of the Struve function differs depending on the values of n. It is found that lower order Struve function gives rise to sinusoidal wave character. We have computed the function $H_n(z)$ for $n=1.5$ and for different values of z. However, in order to maintain the agreement with the actual units of measurement, each value of $H_n(z)$ is multiplied by a constant parameter e^{-12}. The dotted curve which has been drawn on the basis of the expression $e^{-12}H_n(z)$ fits in more or less closely with the

experimental (solid) curve in Fig. 1. It may be noted that the maxima and minima of theoretically computed curve simulate the corresponding values of the experimental curve.

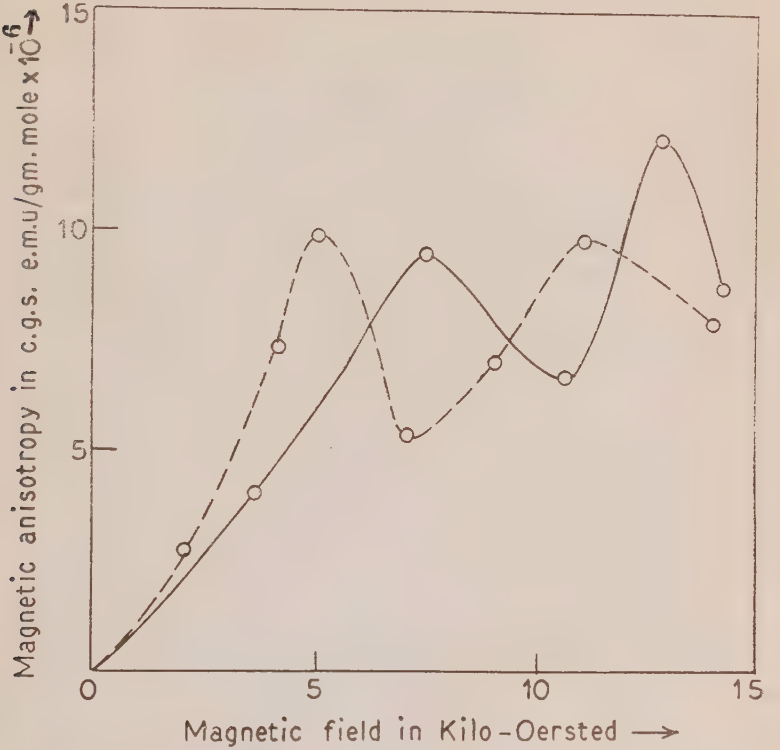

Fig. 1. theoretically computed curve and experimental curve

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INDIAN JOURNAL OF THEORETICAL PHYSICS
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LETTER TO THE EDITOR

On Newton's law of gravitation

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Newton's aim behind the formulation of his theory of gravitation was to give the theoretical basis for Kepler's observation of planetary motion, using his own laws of motion. Although the theory is of overwhelming success and importance, and though it has withstood a time-test of over three hundred years, it is incomplete at least in one respect. It is assumed that the gravitational force of attraction between the sun and the planet provides the centripetal force, necessary to keep the planet orbiting around the sun. This assumption gives rise to the following question. In case of the gravitational potential energy expression of a system of two particles, the same force brings about the displacement of a particle totally different from the planetary motion. Thus when the same force can manifest itself into two different forms in two different situations, we must impose a condition which will allow manifestation of only one kind suitable for that situation. Without such a condition the study of the system would be incomplete.

Ironically the answer to this difficulty seems to be hidden in Newton's law itself. In the previous communications (Sathe 1976 a, b) this law has been considered as made up of two parts, the mass effect and the distance effect-first being the origin of the attractive force and second being that of the repulsive force. The gravitational force has been regarded as the resultant of these forces, and since these forces exist simultaneously ; the concept has been called the "attraction repulsion dualism", The purpose of this communication is to show how the introduction of the repulsive force in Newton's law can be considered as the modified D'Alembertian view.

The repulsive force

The gravitational force is always attractive aud it is very easy to conceive that the mass gives rise to this force. However there is no repulsive force in Newton's law, but how it can be conceived and how to find its origin is explained below.

According to the D'Alembertian view the inertial force always opposes the action of the actual force, keeping the system in equilibrium even in the acceleration. Since it is a natural tendency to acquire the equilibrium state, the inertial force is considered as having the governing role. This new assignment to the inertial force makes Newton's law potent to provide the basis for the justification of the introduction of the repulsive force in Newton's law. In the light of this modified view of D'Alembert, the actual force is attractive and originates in the mass part, whereas the inertial force is repulsive which originates in the distance part. In the present view it is explicitly assumed that the inertial force has origin, in contrast with the D'Alembertian view, and so it is described as the modified D'Alembertian view. In the previous communications the qualitative explanations for the following facts have been given : why the hydrogen forms the predominant part of the universe ?, why two nucleons exist together in the atomic nucleus ?. The explanation for the origin of the spin, which can be applied to the nuclear spin and the planetary spin with equal ease, will be described subsequently.

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INDIAN JOURNAL OF THEORETICAL PHYSICS
SILVER JUBILEE VOLUME (1953-1977)

Torsional vibration of an isotropic circular cylinder of variable rigidity, one end of which is fixed and the other end is subjected to a periodic shearing stress

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[**Abstract :** In this paper the problem of torsional vibration of a finite isotropic circular cylinder possessing a type of non-homogeneity characterised by a parabolic variation of the modulus of rigidity with respect to the axial distance along the cylinder while the material density remains invariable has been solved with prescribed end conditions. Also numerical results have been obtained for the twisting couple acting on a plane end in some particular cases.]

Introduction

Solution of the problem of torsional vibration of homogeneous isotropic circular cylinder is well-known[1]. The problem of torsional vibration of isotropic cylindrical bars has been investigated by Mitra[2] (1961) by considering a type of non-homogeneity in which both μ (the modulus of rigidity) and ρ (the material density) have been made to vary linearly with respect to the axial distance along the bar. The initial aim of this paper was to search for other possible types of variations of μ with respect to the axial distance along the bar by keeping ρ uniform ; and it is found that if the non-homogeneity is characterised by a parabolic rigidity distribution of the form $\mu=\mu_0(kz+1)^2$ where k is a constant and μ_0 is the constant modulus of rigidity on the plane end $z=0$, while ρ remains invariable, the problem of torsional vibration of the cylinder of finite length with prescribed end conditions leads to easy solution. This problem has been

solved in this paper on the assumption that while one end of the cylinder is kept fixed the other end is acted on by a periodic shearing stress. Numerical results have also been obtained giving the magnitude of the twisting couple acting on the fixed end In some particular cases. It is believed that this problem has not been explored by any previous investigator.

Formulation of the problem

We take the axis of the cylinder as the z-axis with the plane end $z=l$, where l is the finite length of the cylinder, as fixed and the origin at the centre of the other end $z=0$. Let a be the radius of a cross section. Introducing cylindrical co-ordinates (r, θ, z) and denoting the components of displacements in the directions r, θ, z by u, v and w respectively we have for this problem

$$u=w=0 \text{ and } v(\neq 0) \text{ is independent of } \theta \quad \cdots \quad (1)$$

So the strain components are

$$\epsilon_r=\epsilon_\theta=\epsilon_z=\gamma_{rz}=0,\ \gamma_{r\theta}=\frac{\partial v}{\partial r}-\frac{v}{r},\ \gamma_{\theta z}=\frac{\partial v}{\partial z} \quad \cdots \quad (2)$$

Hence of all the stress components only $\tau_{r\theta}$ and $\tau_{\theta z}$ are different from zero and these are given by

$$\tau_{r\theta}=\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right),\ \tau_{\theta z}=\mu\frac{\partial v}{\partial z} \quad \cdots \quad (3)$$

where μ is the modulus of rigidity.

Of the three stress equations of motion[3] two are identically satisfied and the third gives

$$\frac{\partial \tau_{r\theta}}{\partial r}+\frac{\partial \tau_{\theta z}}{\partial z}+\frac{2}{r}\tau_{r\theta}=\rho\frac{\partial^2 v}{\partial t^2} \quad \cdots \quad (4)$$

where ρ is the material density.

Substitution of (3) in (4) yields

$$\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)+\frac{\partial}{\partial r}\left[\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)\right]+\frac{2\mu}{r}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)=\rho\frac{\partial^2 v}{\partial t^2} \quad \cdots \quad (5)$$

where
$$\left.\begin{aligned}\mu&=\mu_0(kz+1)^2\\ \rho&=\text{constant}=\rho_0 \text{ say}\end{aligned}\right\} \quad \cdots \quad (6)$$

in which k is a constant.

Let $$v=rf(z)e^{ipt} \quad \cdots \quad (7)$$

This assumption makes the curved surface $r=a$ stress-free.

Substituting (6) and (7) in equation (5) we obtain

$$(kz+1)^2\frac{d^2f}{dz^2}+2k(kz+1)\frac{df}{dz}+m^2f=0 \quad \cdots \quad (8)$$

where
$$m^2=\frac{p^2\rho_0}{\mu_0} \quad \cdots \quad (9)$$

Put
$$kz+1=e^x \quad \cdots \quad (10)$$

Then equation (8) reduces to the simple form

$$\frac{d^2f}{dx^2}+\frac{df}{dx}+\frac{m^2}{k^2}f=0 \quad \cdots \quad (11)$$

whose solution is given by $f=Ae^{\lambda_1x}+Be^{\lambda_2x}$

or by
$$f(z)=A(kz+1)^{\lambda_1}+B(kz+1)^{\lambda_2} \quad \cdots \quad (12)$$

where
$$\lambda_1=\frac{-k+\sqrt{k^2-4m^2}}{2k},\ \lambda_2=\frac{-k-\sqrt{k^2-4m^2}}{2k} \quad (13)$$

when $k>2m$ or by

$$f(z)=\frac{1}{\sqrt{kz+1}}\left\{C_0+C_1\log(kz+1)\right\} \quad \cdots \quad (14)$$

when $k=2m$

Hence relation (7) takes the form

$$v=\frac{r}{\sqrt{kz+1}}\left\{C_0+C_1\log(kz+1)\right\}e^{ipt},\ \text{when}\ k=2m \quad \cdots \quad (15a)$$

and
$$v=r\,[A(kz+1)^{\lambda_1}+B(kz+1)^{\lambda_2}]\,e^{ipt},\ \text{when}\ k>2m \quad \cdots \quad (15b)$$

where C_0, C_1, A, B are arbitrary constants which are to be determined from the following boundary conditions :

$$\left.\begin{aligned} v&=0 \quad \text{at} \quad z=l \\ \mu\frac{\partial v}{\partial z}&=-Sre^{ipt}\ \text{at} \quad z=0 \end{aligned}\right\} \quad \cdots \quad (16)$$

where S is a constant.

Solution of the problem

Case I. $k=2m$. In this case conditions (16) and (15a) yield

$$C_0=\frac{S\log(kl+1)}{\mu_0k\{k+\log(kl+1)\}}$$

$$C_1=\frac{-S}{\mu_0k\{k+\log(kl+1)\}}$$

Hence finally

$$v=\frac{rS\ e^{ipt}}{\mu_0 k\sqrt{kz+1}\{k+\log(kl+1)\}}\log\frac{kl+1}{kz+1} \quad \cdots \quad (17)$$

and the frequency equation is given by

$$p=\frac{k}{2}\sqrt{\mu_0/\rho_0} \quad \cdots \quad (18)$$

Also $$\left[\mu\frac{\partial v}{\partial z}\right]_{z=l}=\frac{-krS\ e^{ipt}}{k+\log(kl+1)} \quad \cdots \quad (19)$$

Numerical calculation

Take $l=10$ cm., $k=1$, $m=\frac{1}{2}$, $a=$radius of the cross section.

$$\text{Magnitude of }\left[\mu\frac{\partial v}{\partial z}\right]_{l=10\text{ cm.}}=rSe^{ipt}\times 0{\cdot}490$$

Therefore magnitude of the twisting couple acting on the section $z=l=10$ cm. is

$$\frac{\pi}{2}\times Sa^4e^{ipt}\times 0{\cdot}490=\pi Sa^4e^{ipt}\times 0{\cdot}245 \qquad \text{(approx.)}$$

Case II. $k>2m$. In this case boundary conditions (16) and 15(b) give

$$A=\frac{-S}{\mu_0 k\left\{\lambda_1-\lambda_2(kl+1)^{\lambda_1-\lambda_2}\right\}}$$

$$B=\frac{S(kl+1)^{\lambda_1-\lambda_2}}{\mu_0 k\left\{\lambda_1-\lambda_2(kl+1)^{\lambda_1-\lambda_2}\right\}}$$

Hence finally

$$v=\frac{rSe^{ipt}}{\mu_0 k\left\{\lambda_1-\lambda_2(kl+1)^{\lambda_1-\lambda_2}\right\}}\times$$
$$\left[(kz+1)^{\lambda_2}(kl+1)^{\lambda_1-\lambda_2}-(kz+1)^{\lambda_1}\right] \quad (20)$$

Therefore,

$$\left(\mu\frac{\partial v}{\partial z}\right)_{z=l}=\frac{rSe^{ipt}(kl+1)^{\lambda_1+1}(\lambda_2-\lambda_1)}{\left[\lambda_1-\lambda_2(kl+1)^{\lambda_1-\lambda_2}\right]} \quad \cdots \quad (21)$$

where $$\lambda_1-\lambda_2=\frac{\sqrt{k^2-4m^2}}{k} \quad \cdots \quad (22)$$

Numerical calculation

Take $l=10$ cm., $k=1$, $m=\frac{1}{4}$, $a=$radius of cross section.

Magnitude of $\left(\mu\frac{\partial v}{\partial z}\right)_{l=10\text{ cm.}}=rSe^{ipt}\times 1\cdot 1$

Therefore magnitude of the twisting couple acting on the section $z=l=10$ cm. is $\pi\ Sa^4e^{ipt}\times 0\cdot 55$ (approx.)

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INDIAN JOURNAL OF THEORETICAL PHYSICS
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Distribution of isotopes among the elements

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[**Abstract :** It is shown that there is a systematic relation among the isotopes of elements.]

It has been shown that the atomic weights and atomic numbers may be related by a single equation so far a curve fitting relation may hold good[1,2,3]. Since atomic weights depend on the number of isotopes it will be interesting to study the distribution of isotopes among the elements found in nature[3].

At first sight it appears that the distribution of isotopes are very irregular. But when one plots the number of isotopes (including unstable elements) against the number of elements (Fig. 1), one finds that curve emerges as a bimodal curve with an approximately regular distribution. Table 1 shows how the total number of isotopes is distributed among the elements[4].

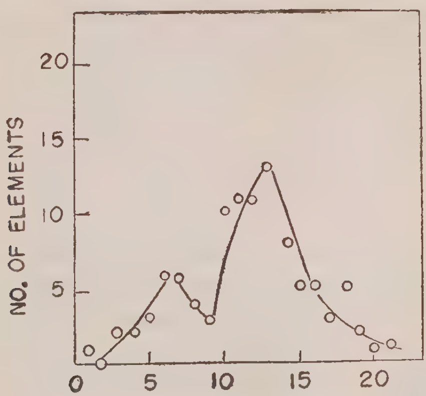

Fig. 1
No. of isotopes

Table 1

No. of isotopes :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
No. of elements :	1	0	2	2	3	6	6	4	3	10/11	11	13	8	5	5	3	5	2	1	1	

For stable isotopes Fig. 2 is a diagrammatic representation of data in Table 2.

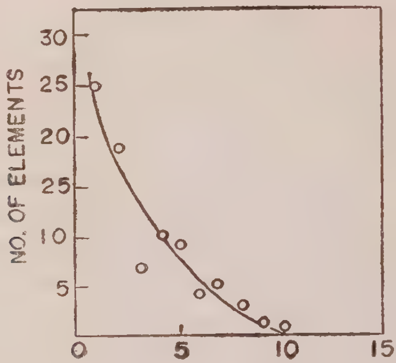

Fig. 2
No. of isotopes

Table 2

No. of isotopes.	1	2	3	4	5	6	7	8	9	10
No. of elements.	25	18	7	10	9	4	5	3	1	1

It seems that the curve has a regular shape.

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INDIAN JOURNAL OF THEORETICAL PHYSICS
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A note on the relativistic corrections to the multiparticle systems

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[**Abstract :** The application of Shirokov's relativistic correction to the two-body potential, on the binding energies of ^3H, ^3He, ^4He and p-d scattering lengths, as formulated by Dhara, has been explored.]

During the last few years, many sets of equations have been proposed for solving the Faddeev non-relativistic scattering three- and four-body problems[1,2,3] using improved potentials in succession. One computes the off-shell three-body scattering amplitudes and the bound states, when one knows the off-shell two-body amplitudes. The most interesting three-body problems are usually involved with resonances of three elementary particles.

Since the relativistic correction to the nuclear-potentials happens to be comparable to the core effect, the effect on three nucleon parameters has to be estimated with sufficient accuracy. The problem of correction to the kinetic energies of the particles is generally linked with the bigger issue of the relativistic dynamics involving two and many-body systems. Though various significant attempts[4] have been made from time to time, no full fledged relativistic dynamics has been developed yet. This note is an attempt to search for a relativistic correction model of the Faddeev equations. The relativistic corrections to certain three-and many-body parameters appear to be realistic so long as the results depend on the totally symmetric part of their wave functions.

Considering the non-relativistic two-body free Hamiltonian in the c.m. system, one defines $\mathbf{p}_1, \mathbf{p}_2$ to be the momenta of particles 1 and 2 respectively in this system. It is also defined that $\mathbf{p}=\mathbf{p}_1+\mathbf{p}_2$ and $\mathbf{p}'=\mathbf{p}_1'+\mathbf{p}_2'$ for initial and final momenta respectively. Then one has

$$<\mathbf{p} \mid H_0 \mid \mathbf{p}'>=\mathbf{p}^2\delta(\mathbf{p}-\mathbf{p}'). \qquad \cdots \qquad (1)$$

The total two-particle Hamiltonian is given by

$$H=H_0+H_{12}, \qquad \cdots \qquad (2)$$

where

$$H_0=\frac{\mathbf{p}_1{}^2}{2m_1}+\frac{\mathbf{p}_2{}^2}{2m_2}. \qquad \cdots \qquad (3)$$

In general, H_{12} must not depend on the total momentum $\mathbf{p}$ as a consequence of conservation law for the non-relativistic centre of inertia (Galilean invariant). The inclusion of the relativistic effect leads to the correction terms which depend on $\mathbf{p}$.

For a two-body potential V involving two equal masses (M), Shirokov[5] expressed the Hamiltonian as

$$H=\frac{\mathbf{p}_1{}^2}{2m_1}+\frac{\mathbf{p}_2{}^2}{2m_2}-\frac{\mathbf{p}_1{}^4}{8m_1{}^3}-\frac{\mathbf{p}_2{}^4}{8m_1{}^3}+H_{12}+H'_{12}, \qquad \cdots \qquad (4)$$

where H'_{12} is the relativistic correction term.

The relativistic correction can be avoided by adopting the centre of inertia system in the two-body problem. The correction becomes significant only in the many-body problem. This relativistic correction does not contain all corrections. These are only those which violate the Galilean invariance of the Hamiltonian. The total Hamiltonian can be used for investigation of the interaction between the high energy nucleons with other nuclei. On the other hand, the effect of relativistic corrections of the structure of the nucleus can be estimated by this Hamiltonian.

For applying the relativistic correction δV to the nucleon-nucleon potential V, one may consider an expression given by Shirokov[5], using the requirement of invariance under the Lorentz

group upto terms of $O(v^2/c^2)$. An interesting feature of this correction is that it is proportional to the square of the c.m. momentum p of the N-N system, so that it vanishes in exactly in the c.m. frame. Since it is usual in most $3N$ calculations to parametrize the N-N potential directly from the data, the last feature of δV which leads only to off c.m. effect $(\mathbf{p} \neq o)$ can have manifestations in three or more-body systems without requiring any relativistic modification in the parameters of the two-body potential V which are supposed to have already been determined from the N-N data in their own c.m. frame $(\mathbf{p} \neq o)$. The Shirokov expression for the second order relativistic correction δV to a two-body potential V for equal masses (M) is given by

$$< \mathbf{P}_p \mid \delta V \mid \mathbf{P}'\mathbf{p}' >$$
$$= -\tfrac{1}{4} M^{-2} \left[P^2 + \tfrac{1}{2}(\mathbf{p}\,\mathbf{P}) \left(\mathbf{P} . \frac{\partial}{\partial \mathbf{p}} \right) + \tfrac{1}{2}(\mathbf{p}'.\mathbf{P}) \left(\mathbf{P} . \frac{\partial}{\partial \mathbf{p}} \right) \right]$$
$$< \mathbf{p} \mid V \mid \mathbf{p}' >, \quad \ldots \qquad (5)$$

where $\mathbf{p}$ and $\mathbf{p}'$ represent the relative momenta and $\mathbf{P}$ is the total c. m. momentum of a two-nucleon system. The kinetic energy correction for a particle of momentum $\mathbf{P}_i$ is

$$\Delta T_i = -\tfrac{1}{8} P_i^4 M^{-3}. \quad \ldots \qquad (6)$$

For the non-relativistic potential V in the c.m. frame we take the s-wave separable form[6] as

$$< \mathbf{p} \mid V \mid \mathbf{p}' > = -\lambda M^{-1} g(\mathbf{p}) g(\mathbf{p}'), \quad \ldots \qquad (7)$$

where

$$g(\mathbf{p}) = (p^2 + \beta^2)^{-1}.$$

Since the two-body potential is separable, it is seen from Eq. (5) that the correction term δV is also the sum of separable, so that it admits the standard manipulation in order to solve the three and four-body equations previously formulated by Dhara[3,7,8,9].

Dhara's formulations of the Faddeev non-relativistic equations for the binding energies of ^3He, ^4He, ^3H and p-d scattering length are presented here without details as

(*i*) the equation for the case of the binding energy of ^3He (Eq. (39) of ref. (7))

$$X_i(E_i', E)=2\sum_{i\neq j}\int Z_{ij}(E_i', E_i'')Z_{ji}(E_i', E_i'')X_i(E_i'', E)dE_i'' \qquad \ldots \quad (8)$$

(*ii*) the equation for the case of the binding energy of the triton (Eq. (9) of ref. (8))

$$X_i(E_i', E)=Y_i(E_i', E)-\sum_j\int Z_{ij}(E_i', E_i'')X_j(E_i'', E)dE_i''. \qquad \ldots \quad (9)$$

(*iii*) the equation for the case of *p-d* scattering length (Eq. (38) of ref. (9))

$$X_i(E_i', E)=\sum_i U_i(E_i', E)-\sum Z_{ij}(E_i', E)U_j(E_i', E)$$

$$+2\sum_{i\neq j}\int Z_{ij}(E_{ij}'E_i'')Z_{jj}(E_i', E_i'')X_i(E_i'', E)dE_i''\ldots \qquad (10)$$

where $Z_{ij}=0$, if $i=j (i, j=1, 2, 3)$.

(*iv*) the equation for the case of the binding energy of ^4He (Eq. (21) of ref. (3))

$$\begin{pmatrix} T^1(z)\\ T^2(z)\\ T^3(z)\\ T^4(z)\\ T^5(z)\\ T^6(z)\end{pmatrix}=\begin{pmatrix} T^c_{12}(z)\\ T_{13}(z)\\ T_{14}(z)\\ T_{23}(z)\\ T_{24}(z)\\ T_{34}(z)\end{pmatrix}-\begin{pmatrix} 0 & T^c_{12}(z) & T^c_{12}(z) & T^c_{12}(z) & T^c_{12}(z) & T^c_{12}(z)\\ T_{13}(z) & 0 & T_{13}(z) & T_{13}(z) & T_{13}(z) & T_{13}(z)\\ T_{14}(z) & T_{14}(z) & 0 & T_{14}(z) & T_{14}(z) & T_{14}(z)\\ T_{22}(z) & T_{23}(z) & T_{23}(z) & 0 & T_{23}(z) & T_{23}(z)\\ T_{24}(z) & T_{24}(z) & T_{24}(z) & T_{24}(z) & 0 & T_{24}(z)\\ T_{34}(z) & T_{34}(z) & T_{34}(z) & T_{34}(z) & T_{34}(z) & 0\end{pmatrix} G_o^c(z)\begin{pmatrix} T^1(z)\\ T^2(z)\\ T^3(z)\\ T^4(z)\\ T^5(z)\\ T^6(z)\end{pmatrix} \qquad \ldots \quad (11)$$

The author wishes to discuss these features in details with numerical results in a future communication. Incidentally, Bhasin et al[10] have got some interesting result indicating 5% decrease in the binding energy of the triton due to the three-body effect of this correction.

The author is grateful to Professor S. D. Chatterjee for his interest in this problem.

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INDIAN JOURNAL OF THEORETICAL PHYSICS
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A Mandelstam representation for nucleonic form factors

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[**Abstract :** A new shape to the dispersion theoretic analysis of form factors of nucleons is given on the basis of a lepton-hadron model of strong interactions as posited by P. Bandyopadhyay. The model which rules out $(\rho-\omega)$ mixing and reposes near-full confidence in ρ-dominance in $(\pi-\pi)$ processes, admits of a double dispersion relation for nucleon form factors where, of course, the ρ is described by a Yukawa class of potential.]

Introduction

The $(e-p)$ Scattering is the convenient probe for the study of $e-m$ from factor of the proton. Also, the Rosenbluth formula is good enough until a momentum transfer of $|t| \approx 1 \cdot 3\ (Gev/c)^2$ is crossed. Our concern is the large $|t|$ region where strong interactions dominate. Strong interactions can release such particles or resoncences as would interact electromagnetically with the proton (we shall confine ourselves to the proton although the generic term 'nucleon' would be used). And the Bandyopadhyay parton model[1] that effectively reduces nucleonic processes to pionic interactions paves the way for the introduction of the double spectral function. This is achieved very simply since unlike in other approaches where the nucleon is the interacting entity by itself, here we have two pious in the nucleon configuration (apart from another spectator lepton) so that a double scattering process between the exchanged ρ and the π's in the manner of Glauber double impulse is quite fesible.

The unitarity (box) diagram for the process follows easily and this is how a formal incorporation of the double spectral function of Mandelstam in the proton form factor problem is realized here.

The moment we look for working formulas we may invoke the Blankenbeckler-Goldberger pattern[2] of Yukawa-potential for describing the ρ.

Let us note in passing that the authors[3] who have applied the dispersion relations to nucleon form factors have been content with single dispersion relations for form factors.

The usual notion of proportionality between the form factor squared and the scattering cross-section—a pragmatically tenable premissunderlies the present work too. The raison d' etre of this paper is to point to the possibility of a double dispersion theory for the case, in question, the rest of what follows being essentially an exposition, or to use the cliche, a brief review, of the familiar material that a realization of such possibility must utilize.

The introduction of the double spectral function

The relevant part of the diagram with a two-particle intermediate state is given in the unitarity graph of Fig. 1. (Where isospin differentia are ignored for convenience) :

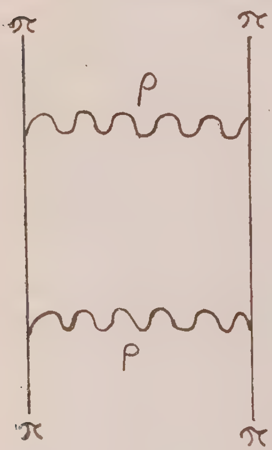


Fig. 1

The underlying scheme of the present approach to double dispersion is provided by the Cutkosky rules[4] for representing the discontinuity across the cut associated with an N particle intermediate state :

$$\text{Disc}_N\,[A]=\int \prod_{i=1}^{N}\left[\frac{id^4k}{(2\pi)^4}\right]\prod_{i=1}^{N}\left[-2\pi i\delta^+\left(q_i^2-m_i^2\right)\right]\times A_1^+A_2^- \quad \ldots \quad (1)$$

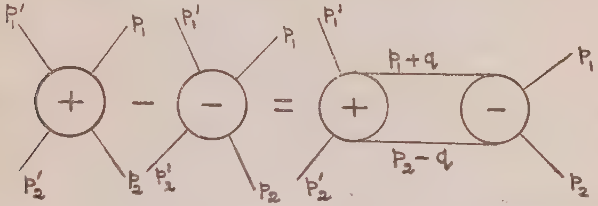
where the notations are as usual.

Feynman rules would have coupling constants in place of the

vertex functions A^+, A^-. Landan rules[5] that dictate the singularities of these integrals have been assiduously examined by the proponents of a unison of the field theory and the dispersion theory.

Form factor investegations, both theoretical and experimental, conclusively point to an edge enjoyed by the dispersion theory. We feel the affected unison view-point is a procrustean bed so far as the departure from Feynman rules as shown by Eq. (1) is at issue.

We can cast our double impulse process into the following two particle unitarity diagram Eq :



.. (2)

To arrive at Mandelstam representation is now a routine work. Instead of reproducing all the intermediate arguments, we wish to exhort certain pertinent points of import.

(*i*) Let S − value be such as would allow a two − particle state alone.

(*ii*) Let the discontinuity across the cuts in S be given by

$$D_s(s, t)=\frac{1}{2i}\Big[A(s+i\varepsilon, t)-A(s-i\varepsilon, t)\Big]$$

(*iii*) Let us use the well-known results of kinematics together with the formula

$$\int d^3\vec{q}=\frac{1}{2}\int\left|\vec{q}\right|\cdot d\left|\vec{q}\right|^2 d\Omega$$

All these being utilized, one obtains :

$$D_s(s, t)=\frac{q_s}{32\pi^2\sqrt{s}}\int d\Omega A^+(s, t')A^-(s, t'') \quad \cdots \quad (3)$$

where t' = final state-intermediate state momentum transfer and t'' = initial state-intermediate state momentum transfer (in the c. m. system).

The maximal analyticity conjecture for the S-matrix requires that the discontinuity in S given by (3) should have a t-discontinuity and the statement is valid for a cyclic permutation of the channels s, t, u. Another conjecture is that all branch points lie on the real axis.

Thus we may define the S-discontinuity of D_t by

$$\rho_{st}(s, t)=\frac{1}{2i}\left[D_t(s+i\varepsilon, t)-D_t'(s-i\varepsilon, t)\right] \quad \cdots \quad (4)$$

$$(s>b_1(t)>0)$$

Neglecting bound-state poles and ignoring the u-channel, one has the unsubtracted fixed S dispersion relation for the single variable :

$$A(s, t, u)=\frac{1}{\pi}\int_{t_0}^{\infty}\frac{dt'}{t'-t}\, D_t(s, t') \quad \cdots \quad (5)$$

By (4) and (5),

$$A(s, t)=\frac{1}{\pi^2}\iint^{\infty}\frac{\rho_{st}(s', t')}{(s'-s)(t'-t)}\, ds'\, dt' \quad \cdots \quad (6)$$

This is the Mandelstam representation (in absence of the u-channel) containing the double spectral function ρ_{st}.

Standard techniques are in vogue for the calculation of the double spectral function. The strip approximation or the Mandelstam iteration may be resorted to for the purpose.

The large S-anomaly for n iterations *viz.*,

$$\rho^{el}(s, t) \sim s^{2n\alpha(t_1)-n}$$

as one puts a t-channel Regge pole $\alpha(t)$ into the part-amplitude that is assumed to be known (which anomaly is related to the $A\ F\ S$ cut) arises in the unfounded conjecture of 'elastic unitarity' even for large S. Mandelstam[6] has shown how delicate cancellations due to many-body intermediate state cuts might set things in order. And thus a Regge asymptotic behaviour is safeguarded. Our problem being essentially of a high energy region, the above remedy of the supposed anomaly is indeed gratifying.

3. *The utilization of the Yukawa potential*

It is well known that the form factor is proportional to the scattering amplitude. We are out to explore the second Born term f_2 by representing the ρ by means of the simple Yukawa potential.

Blankenbecler et al. have shown that with a simple Yukawa potential

$$f_2(s,t)=\int_0^\infty \frac{ds'}{\pi}\int_0^\infty \frac{dt'}{\pi}\,\frac{\rho_2(s',t')}{(s'-s-i\varepsilon)(t'-t)} \quad \cdots \quad (7)$$

with
$$\rho_2(s,t)=\frac{v_0^{\,2}}{2}\,\frac{\pi}{\sqrt{t}}\,\frac{\Theta(st-\mu^4-4s\mu^2)}{\sqrt{st-\mu^4-4s\mu^2}} \quad \cdots \quad (8)$$

where $v_0=2mV_0$ and the potential $V(r)$ is :

$$rV(r)=V_0\int_0^\infty d\mu\,6(\mu)e^{-\mu r} \quad \cdots \quad (9)$$

From (8) the non-vanishing region for ρ_2 is seen to be

$$t>4\mu^2+\frac{\mu^4}{s};\quad \mu=\rho-\text{mass.}$$

As one extends f_2 as a function of two complex variables s, t, the function is analytic in the product of the two cut planes—the s-cut running from 0 to ∞ and the t-cut from $4\mu^2$ to ∞.

Consideriug t real and negative and letting S approach the positive real axis from above would correspond to the physical amplitude.

The S'-integration being carried out, the formula for f_2 as shown in reference 2 reads :

$$f_2(s,t)=\pi v_0^{\,2}\int_{4\mu^2}^{4\mu^2+\frac{\mu^4}{s}} \frac{dt'}{\pi}\,\frac{1}{\sqrt{t'}(t'-t)}\,\frac{1}{\sqrt{\mu^4-(t'-4\mu^2)s}}$$
$$+i\pi v_0^{\,2}\int_{4\mu^2+\frac{\mu^4}{s}}^{\infty} \frac{dt'}{\pi}\,\frac{1}{\sqrt{t'}(t'-t)}\,\frac{1}{\sqrt{(t'-4\mu^2)s-\mu^4}} \quad \cdots \quad (10)$$

The working of these integrals is another matter, which in any case, does not concern us here. We have seen from above how a simple structure—notion 'foisted' on the proton enhances the field of application of the wanton but prestigious 'double spectral function' and that too for the case of what may rightly be called the 'alpha' of the baryons. Who can tell if the 'omega' would simulate the example? The author takes pleasure in thanking P. Bandyopadhyay who evinced a lively interest in the present work and Ruma Basu Roy who made some references available.

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INDIAN JOURNAL OF THEORETICAL PHYSICS
SILVER JUBILEE VOLUME (1953-1977)

Elastic distortion of a non-homogeneous aeolotropic thick-walled cylindrical shell by tangential traction on the outer boundary

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[**Abstract**: The elastic distortion of a hollow circular cylinder of cylindrically aeolotropic and non-homogeneous material by tangential tractions on the outer boundary have been discussed. The inner boundary of the shell is free.]

Introduction

Torsional stresses and deformations of isotropic circular cylinders acted upon by tractions applied on the boundary were determined by L. N. G. Filon[1]; H. Reissner and E. Reissner[2] have discussed the problem for aeolotropic material. Chattarji[3] discussed the twisting of a hollow circular cylinder of cylindrically aeolotropic material with outer surface fixed and inner surface acted on by tangential tractions. Ghosh[4] determined the deformation and stresses of an isotropic non-homogeneous thick walled cylindrical shell by tangential stresses on the outer surface. In the present paper the authors deal with the determination of the deformations and torsional stresses in a hollow circular cylindrical body of non-homogeneous and cylindrically aeolotropic material with outer surface acted on by tangential tractions and inner surface free. The outer radius of the shell is a and the inner radius is b. The elastic constants are taken to vary as the nth power of the radial distance from the axis of the shell.

Formulation of the problem

The equations of equilibrium in cylindrical polar co-ordinates (r, θ, z) in absence of body forces are [Love p. 90][5]

$$\left.\begin{aligned}
&\frac{\partial}{\partial r}\widehat{rr}+\frac{1}{r}\frac{\partial}{\partial\theta}\widehat{r\theta}+\frac{1}{r}(\widehat{rr}-\widehat{\theta\theta})=0\\
&\frac{\partial}{\partial r}\widehat{r\theta}+\frac{1}{r}\frac{\partial}{\partial\theta}\widehat{\theta\theta}+\frac{\partial}{\partial z}\widehat{\theta z}+\frac{2\widehat{r\theta}}{r}=0\\
&\frac{\partial}{\partial r}\widehat{zr}+\frac{1}{r}\frac{\partial}{\partial\theta}\widehat{\theta z}+\frac{\partial}{\partial z}\widehat{zz}+\frac{\widehat{zr}}{r}=0
\end{aligned}\right\} \quad \cdots \quad (1)$$

The stresses are given by,

$$\left.\begin{aligned}
&\widehat{rr}=c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz}\\
&\widehat{\theta\theta}=c_{21}e_{rr}+c_{22}e_{\theta\theta}+c_{23}e_{zz}\\
&\widehat{zz}=c_{31}e_{rr}+c_{32}e_{\theta\theta}+c_{33}e_{zz}\\
&\widehat{\theta z}=c_{44}e_{\theta z},\ \widehat{zr}=c_{55}e_{zr},\ \widehat{r\theta}=c_{66}e_{r\theta}\\
\text{where}\quad &c_{ij}=c_{ji}, \qquad (i, j=1, 2, \cdots\cdots 6).
\end{aligned}\right\} \quad \cdots \quad (2)$$

If u, v, w be the components of the displacement for axially symmetric deformations in cylindrical co-ordinates, the components of strain are given by

$$\left.\begin{aligned}
&e_{rr}=\frac{\partial u}{\partial r},\ e_{\theta\theta}=\frac{u}{r},\ e_{zz}=\frac{\partial w}{\partial z}\\
&e_{r\theta}=\frac{\partial v}{\partial r}-\frac{v}{r},\ e_{\theta z}=\frac{\partial v}{\partial z},\ e_{zr}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}
\end{aligned}\right\} \quad \cdots \quad (3)$$

Taking the axis of the cylindrical shell as the axis of z and assuming $u=w=0$, we find that the stress components are

$$\widehat{rr}=\widehat{\theta\theta}=\widehat{zz}=\widehat{zr}=0$$

$$\widehat{r\theta}=c_{66}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)=c_{66}\, r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)$$

$$\widehat{\theta z}=c_{44}\frac{\partial v}{\partial z}$$

where c_{44} and c_{66} are the two moduli of rigidity. Let us take $c_{ij}=r^s C_{ij}$. Taking v to be independent of θ and functions of r only we find that the first and the third equations of equilibrium are identically satisfied and the second equation reduces to,

$$\frac{\partial^2 v}{\partial r^2}+\frac{s+1}{r}\frac{\partial v}{\partial r}-\frac{s+1}{r^2}v+k^2\frac{\partial^2 v}{\partial z^2}=0 \quad \cdots \quad (4)$$

where $k^2=\frac{C_{44}}{C_{66}}$.

Now substituting $v=R(r)\cos\alpha z$ or $R(r)\sin\alpha z$, the equation (4) reduces to

$$\frac{d^2R}{dr^2}+\frac{s+1}{r}\frac{dR}{dr}-\left(\frac{s+1}{r^2}+k^2\alpha^2\right)R=0 \quad \cdots \quad (5)$$

The solution of the equation (5) is

$$R=(r\alpha)^{-s/2}[A_\alpha K_p(k\alpha r)+B_\alpha I_p(k\alpha r)]$$

where $I_p(k\alpha r)$ and $K_p(k\alpha r)$ are modified Bessel functions of the first kind and the second kind respectively.

Case I. The cylindrical shell is under uniform tangential traction on the outer surface and one end of the shell is fixed while the other end is free.

In this case let us take the length of the cylindrical shell to be L ; the end $z=0$ is kept fixed and $z=L$ is free. The tangential traction is acting throughout the outer surface of the shell and the inner surface is free. Then the boundary conditions are

$$\left.\begin{array}{lll} v=0 & \text{on } z=0 & \\ \widehat{z\theta}=0 & \text{on } z=L & \\ \widehat{r\theta}=-S, & \text{where } 0<z<L & \text{on } r=a \\ \quad =0 & & \text{on } r=b \end{array}\right\} \quad \cdots \quad (6)$$

We take in this case v as,

$$v=\sum_\alpha[A_\alpha K_p(k\alpha r)+B_\alpha I_p(k\alpha r)](r\alpha)^{-s/2}\sin\alpha z \quad \cdots \quad (7)$$

Then the stresses will be obtained as,

$$\left.\begin{aligned}\widehat{r\theta} &= -\sqrt{C_{44}C_{66}}\sum_{\alpha}[A_{\alpha}K_{p+1}(k\alpha r)-B_{\alpha}I_{p+1}(k\alpha r)] \\ &\qquad\times(\alpha r)^{1-s/2}r^{s-1}\sin\alpha z \\ \widehat{\theta z} &= C_{44}\sum_{\alpha}[A_{\alpha}K_{p}(k\alpha r)-B_{\alpha}I_{p}(k\alpha r)] \\ &\qquad\times(\alpha r)^{1-s/2}r^{s-1}\cos\alpha z.\end{aligned}\right\}\quad\cdots\quad(8)$$

The first condition of the equation (6) is evidently satisfied. The second condition of the equation (6) gives

$$\alpha=\frac{(2n+1)\pi}{2L}$$

where n is any integer.

From the other two conditions of the equation (6) we get,

$$A_{\alpha}=\frac{2SI_{p+1}(k\alpha b)}{\alpha L\sqrt{C_{44}C_{66}}(\alpha a)^{1-s/2}a^{s-1}}\times\frac{1}{[K_{p+1}(k\alpha a)I_{p+1}(k\alpha b)-K_{p+1}(k\alpha b)I_{p+1}(k\alpha a)]}$$

$$B_{\alpha}=\frac{2SK_{p+1}(k\alpha b)}{\alpha L\sqrt{C_{44}C_{66}}(\alpha a)^{1-s/2}a^{s-1}}\times\frac{1}{[K_{p+1}(k\alpha a)I_{p+1}(k\alpha b)-K_{p+1}(k\alpha b)I_{p+1}(k\alpha a)]}$$

Hence from the equations (7) and (8) after substituting the values of A_{α} and B_{α} from (9) we get the displacements and stresses as,

$$v=\sum_{\alpha}\frac{2S}{\alpha^{2}L\sqrt{C_{44}C_{66}}(ra)^{s/2}}\left[\frac{I_{p+1}(k\alpha b)K_{p}(k\alpha r)-K_{p+1}(k\alpha b)I_{p}(k\alpha r)}{K_{p+1}(k\alpha a)I_{p+1}(k\alpha b)-K_{p+1}(k\alpha b)I_{p+1}(k\alpha a)}\right]\sin\alpha z$$

$$\widehat{r\theta}=-\sum_{\alpha}\frac{2S(\alpha r)^{s/2}}{\alpha L(\alpha a)^{s/2}}\left[\frac{I_{p+1}(k\alpha b)K_{p+1}(k\alpha r)-K_{p+1}(k\alpha b)I_{p+1}(k\alpha r)}{K_{p+1}(k\alpha a)I_{p+1}(k\alpha b)-K_{p+1}(k\alpha b)I_{p+1}(k\alpha a)}\right]\sin\alpha z$$

$$\widehat{\theta z}=\sum_{\alpha}\frac{2Skr^{s}}{\alpha L(ar)^{s/2}}\left[\frac{I_{p+1}(k\alpha b)K_{p}(kr\alpha)+K_{p+1}(k\alpha b)I_{p}(k\alpha r)}{K_{p+1}(k\alpha a)I_{p+1}(k\alpha b)-K_{p+1}(k\alpha b)I_{p+1}(k\alpha a)}\right]\cos\alpha z$$

Case II. The cylindrical shell is under uniform tangential traction over a portion of its outer surface and the inner surface is free.

In this case we take the length of the cylindrical shell to be $2L$, the ends of the shell are free. The tangential traction is acting within the portion $z=\pm h$. Hence the boundary conditions are,

$$\left.\begin{aligned} &v=0, && \text{on } r=b \\ &\widehat{\theta z}=0, && \text{on } z=\pm L \\ &\widehat{r\theta}=-S_1, \text{ when } |z|<h && \\ &\quad =0, \quad \text{when } |z|>h && \text{on } r=a \end{aligned}\right\} \quad \ldots \quad (9)$$

In this case we take v as

$$v=A_0 r+\frac{B_0}{r^2}+\sum_{\lambda}\Big[C_\lambda K_p(k\lambda r)+D_\lambda I_p(k\lambda r)\Big](r\lambda)^{-s/2}\cos\lambda z.$$

The first condition of the equation (9) gives,

$$v=A_0\left(r-\frac{b^3}{r^2}\right)$$

$$+\sum_{\lambda}C_\lambda\left[\frac{K_p(k\lambda r)I_p(k\lambda b)-I_p(k\lambda r)K_p(k\lambda b)}{I_p(k\lambda b)}\right](r\lambda)^{-s/2}\cos\lambda z.$$

Also from the second condition of the equation (9) we get

$\lambda=\dfrac{n\pi}{L}$ where n is an integer.

From the third boundary condition of equation (9) we get,

$$A_0=-\frac{S_1 h}{3b^3a^{s-3}C_{66}L}$$

and $$C_\lambda=\frac{2S_1\sin\lambda h I_p(k\lambda b)}{\lambda L(\lambda a)^{1-s/2}\sqrt{C_{44}C_{66}}a^{s-1}}$$

$$\times\frac{1}{[K_{p+1}(k\lambda a)I_p(k\lambda b)+I_{p+1}(k\lambda a)K_p(k\lambda b)]}$$

Hence the displacement v on $r=a$ is obtained as,

$$(v)_{r=a}=\frac{S_1 h}{3b^3 a^{s-3} C_{66} L}\left(\frac{b^3}{a^2}-a\right)$$
$$+\sum_\lambda \frac{2S_1 \sin \lambda h}{\lambda L(\lambda a)^{1-s/2} a^{s-1}\sqrt{C_{44}C_{66}}}$$
$$\left[\frac{K_p(k\lambda a)I_p(k\lambda b)-I_p(k\lambda a)K_p(k\lambda b)}{K_{p+1}(k\lambda a)I_p(k\lambda b)+I_{p+1}(k\lambda a)K_p(k\lambda b)}\right]\cos \lambda z\,.$$

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INDIAN JOURNAL OF THEORETICAL PHYSICS
SILVER JUBILEE VOLUME (1953-1977)

Unsteady motion of an infinite viscoelastic liquid due to nonuniform rotation of a sphere

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[**Abstract** : In the present paper the author has discussed unsteady slow flow of an infinite Rivilin-Ericksen[6] viscoelastic liquid due to the nonuniform rotation of a smooth and rough sphere separately, about z-axis (taken as axis of rotation), under an external force $z\omega^2$ acting per unit mass of the liquid along the axis of rotation. The result obtained is also valid for the slow rotation of the sphere. In that case the external force does not exist and the pressure remains constant throughout the liquid, In this paper first of all the exact solution for the problem has been obtained when the sphere is smooth and later on the problem has been solved for the rough sphere using Stokes approximation. Here it has been assumed that the roughness is small compared to the radius of the smooth sphere and the results have been calculated to the first power of ϵ, where ϵ is the roughness parameter.]

Introduction

The steady flow of viscous liquid due to the slow rotation of a smooth sphere has already been studied by Lamb[5]. Ghildyal[3] studied the unsteady motion of an infinite liquid due to the uniform rotation of a smooth sphere $r=a$. Gupta and Kulsrestha[4] have discussed unsteady slow viscous flow of an infinite liquid due to the nonuniform rotation of a smooth and rough sphere seperately, about z-axis, under an external force $z\omega^2$ acting per unit mass of the liquid along the axis of rotation.

In the present paper the author has considered unsteady slow flow of an infinite Rivilin-Ericksen[6] viscoelastic liquid due to nonuniform rotation of a smooth and rough sphere separately,

about z-axis (taken as axis of rotation), under an external force $z\omega^2$ acting per unit mass of the liquid along the axis of rotation and the pressure at any point is given by

$$p=\int \rho r\omega^2 dr + a \text{ constant}, \quad \cdots \quad (1)$$

where $\omega=\omega(r, t)$, $r^2=x^2+y^2+z^2$ and velocity components at any point (x, y, z) of the liquid are assumed to be

$$u=-\omega y,\ v=\omega x \text{ and } W=0.$$

Basic equation of motion

This paper treats Rivilin-Ericksen viscoelastic liquid. The constitutive equation is (Siddappa,[7])

$$T=-pI+\phi_1\, d+\phi_2\, b+\phi_3\, d^2, \quad \cdots \quad (2)$$

where $T=\| T_{ij} \|$, $I=\| \delta_{ij} \|$, δ_{ij}=Kronecker delta,

$d=\| d_{ij} \|$, $d_{ij}=\frac{1}{2}(W_{i,\, j}+W_{j,\, i})$=Deformation rate tensor

$b=\| b_{ij} \|$, $b_{ij}=a_{i,\, j}+a_{j,\, i}+2W_{m,\, i}W^m{}_{i,\, j}$=viscoelastic tensor;

with $\bar{a}_i=\dfrac{\partial W_i}{\partial t}+W_{i,\, j}\, W^j$=Acceleration vector.

The ϕ_1, ϕ_2, ϕ_3 are co-efficients of viscosity, viscoelasticity, cross-viscosity respectively and these are in general functions of the temperature, material properties and invariants of d, b, d^2.

Te basic equations of motion are reduced to

$$\frac{\partial \omega}{\partial t}=-\frac{1}{\rho}\frac{\partial \rho}{\partial t}+\left(\alpha+\beta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}+\frac{\partial^2 \omega}{\partial z^2}\right), \quad \cdots \quad (3)$$

$$\frac{\partial p}{\partial r}=\rho r\omega^2, \quad \cdots \quad (4)$$

where $-\dfrac{1}{\rho}\dfrac{\partial \rho}{\partial t}$ is pressure gradient, α the Kinematic viscosity, β Kinematic viscoelasticity and ω is the angular velocity of rotation and $\omega=\omega(r, t)$.

But in case of viscoelastic homogeneous incompressible liquid $\left(\frac{\partial \rho}{\partial t}=0\right)$ above equation (3) (in spherical polar co-ordinates) reduces to

$$\frac{\partial \omega}{\partial t}=\left(\alpha+\beta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 \omega}{\partial r^2}+\frac{4}{r}\frac{\partial \omega}{\partial r}\right) \quad \cdots \quad (5)$$

Let $r(z)$ be the radius of the sphere, where

$$r(z)=a+\epsilon N(z) \quad \cdots \quad (6)$$

where 'a' is the radius of smooth sphere.

Let Ω be the initial angular velocity of the sphere. The boundary conditions are then

$$\omega=\Omega r(z)e^{i\alpha\sigma t} \text{ at } r=r(z) \quad \cdots \quad (7)$$

and $\omega=0$ at $r=\infty$ $\cdots$ (8)

for all time t.

Clearly the rotation of the sphere is nonuniform and oscillatory type due to the perturbation term $e^{i\sigma\alpha t}$.

Solution for smooth sphere

If we substitute $\alpha t=T$ and $\omega(r, t)=r^{-\frac{3}{2}}$. $W(r, T)$ in the equation (5) and the boundary conditions (7) and (8), we get

$$\frac{\partial W}{\partial T}=\left(1+\beta\frac{\partial}{\partial T}\right)\left(\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}-\frac{9}{4}\frac{W}{r^2}\right) \quad \cdots \quad (9)$$

and $W=\Omega a^{\frac{5}{2}}e^{i\sigma T}$ when $r=a$, since for smooth sphere

$$r(z)=a \quad \cdots \quad (10)$$

$W=0$ when $r=\infty$ $\cdots$ (11)

for all time t.

Let us now assume

$$W(r, T)=X(r)e^{i\eta T} \quad \cdots \quad (12)$$

then

$$\frac{d^2 X}{dr^2}+\frac{1}{r}\frac{dX}{dr}-\left(\lambda^2+\frac{9}{4r^2}\right)X=0 \quad \cdots \quad (13)$$

where $\lambda^2=i\eta/(1+\beta i\eta)$

The complete solution of (13) is

$$X(r)=AI_{\frac{3}{2}}(\lambda r)+BK_{\frac{3}{2}}(\lambda r) \quad \cdots \quad (14)$$

where $I_{\frac{3}{2}}(\lambda r)$ and $K_{\frac{3}{2}}(\lambda r)$ are the modified Bessel functions of first and second kind respectively and A, B are constants.

From the condition at infinity, we get

$$A=0 \quad \cdots \quad (15)$$

Using the boundary condition (10), we get

$$B=\frac{\Omega a^{\frac{5}{2}}(1-\beta\lambda^2)}{\{i\sigma(1-\beta\lambda^2)-\lambda^2\}K_{\frac{3}{2}}(\lambda a)} \quad \cdots \quad (16)$$

Substituting the values of A and B in (14), we get

$$X(r)=\frac{\Omega a^{\frac{5}{2}}(1-\beta\lambda^2)K_{\frac{3}{2}}(\lambda r)}{\{i\sigma(1-\beta\lambda^2)-\lambda^2\}K_{\frac{3}{2}}(a\lambda)} \quad \cdots \quad (17)$$

Therefore,

$$W(r, T)Re.\left[\frac{\Omega a^{\frac{5}{2}}(1-\beta\lambda^2)\,K_{\frac{3}{2}}(\lambda_r)}{\{i\delta(1-\beta\lambda^2)-\lambda^2\}K_{\frac{3}{2}}(a\lambda)},\ e^{i\eta T}\right] \quad \cdots \quad (18)$$

On substituting back $W(r, T,=r^{\frac{3}{2}}.\omega'r, t)$ and $T=at$, we get

$$\omega(r, t)=Re.\left[\frac{\Omega a^{\frac{5}{2}}(1-\beta\lambda^2)r^{-\frac{3}{2}}K_{\frac{3}{2}}(\lambda r)}{\{i\sigma(1-\beta\lambda^2)-\lambda^2\}K_{\frac{3}{2}}(a\lambda)},\ e^{ia\eta T}\right] \quad \cdots \quad (19)$$

To evaluate pressure, we have from (4)

$$\int_{p_a}^{p_0} dp=Re.\,\rho\int_a^r r\omega^2 dr=\rho\cos(2a\eta t)\int_a^r \frac{B^2}{r^2}\,K^2_{\frac{3}{2}}(\lambda r)dr$$

or,

$$p_0=p_a+\frac{\lambda\beta^2\rho}{4\sqrt{\pi}}\cosh\left(\frac{2a\lambda^2 t}{\beta\lambda^2-1}\right)$$

$$\left[G^{31}_{24}\left(\lambda^2 r^2\middle|\begin{matrix}1, 0\\ 1, -2, -\frac{1}{2}, 0\end{matrix}\right)-G^{31}_{24}\left(a^2\lambda^2\middle|\begin{matrix}1, 0\\ 1, -2, -\frac{1}{2}, 0\end{matrix}\right)\right]$$

by virtue of the result given in Table of Integral Transforms by Erdelyi[2], Vol. 2, 1954, p. 442. Here $G^{mn}_{pq}\left(z \,\middle|\, \begin{matrix} a_1,\cdots,a_p \\ b_1,\cdots,b_q \end{matrix}\right)$ is Meijer's G function and B is the constant given by (16) ; $p_a=p_o$ when $r=a$.

Solution for rough sphere

In this case the roughness of the sphere can be sinusodial roughness, *i.e.* if $N(z)=\sin \frac{z}{l}$, then roughness wavelength will be equal to $2\pi l$.

The boundary conditions are :

$$W=\Omega\{r(z)\}^{\frac{5}{2}}e^{i\sigma at} \qquad \text{when } r=r(z)=a+\epsilon N(z) \;\cdots \tag{20}$$

$$W=0 \qquad \text{when } r=\infty \;\cdots \tag{21}$$

for all time t.

Now multiplying both sides of the equation (9) by $e^{-i\eta T}$ and integrating with respect to T within the limits $T=-\infty$ to $T=\infty$, we have

$$\frac{d^2y}{dr^2}+\frac{1}{r}\frac{dy}{dr}-\left(\lambda^2+\frac{9}{4r^2}\right)y=0 \;\cdots \tag{22}$$

$$\text{where} \quad y=\int_{-\infty}^{\infty} e^{-i\eta T}.W(r,T)dT \text{ and } \lambda^2=i\eta/(1+i\beta\eta)\cdots \tag{23}$$

The complete solution of (22) is given by

$$y=CI_{\frac{3}{2}}(\lambda r)+DK_{\frac{3}{2}}(\lambda r) \;\cdots \tag{24}$$

Using the condition at infinity, we have

$$C=0 \;\cdots \tag{25}$$

Hence $y=D\,K_{\frac{3}{2}}(\lambda r)$ $\cdots$ (26)

Using the boundary condition (20), we have

$$\frac{\Omega[a+\epsilon N(z)]^{\frac{5}{2}}.(1-\beta\lambda^2)}{\{i\sigma(1-\beta\lambda^2)-\lambda^2\}}=DK_{\frac{3}{2}}[\lambda(a+\epsilon N(z))] \;\cdots \tag{27}$$

We now expand functions appearing in the equation (27) in powers of small parameter ϵ and equate like powers of ϵ^0 and ϵ.

Let us assume

$$D = D_0 + \epsilon D_1 + \epsilon^2 D_2 + \cdots\cdots \qquad \cdots \quad (28)$$

Using the following,

$$K_{\frac{3}{2}}\{\lambda(a+\epsilon N(z))\} = K_{\frac{3}{2}}(a\lambda) + a\epsilon N(z) \left\{\frac{3}{2a\lambda} K_{\frac{3}{2}}(a\lambda) - K_{\frac{1}{2}}(a\lambda)\right\} \qquad \cdots \quad (29)$$

We get

$$D_0 = \frac{\Omega a^{\frac{5}{2}}(1-\beta\lambda^2)}{\{i\sigma(1-\beta\lambda^2) - \lambda^2\} K_{\frac{3}{2}}(a\lambda)} \qquad \cdots \quad (30)$$

$$D_1 = \frac{\Omega a^{\frac{5}{2}} N(z)(1-\beta\lambda^2)}{\{i\sigma(1-\beta\lambda^2) - \lambda^2\} K_{\frac{3}{2}}(a\lambda)} \left\{\frac{5}{2} + \frac{2a\lambda K_{\frac{1}{2}}'(a\lambda) + 3K_{\frac{3}{2}}(a\lambda)}{2K_{\frac{3}{2}}(a\lambda)}\right\} \qquad \cdots \quad (31)$$

Therefore,

$$y = \frac{\Omega a^{\frac{5}{2}}(1-\beta\lambda^2) K_{\frac{3}{2}}(\lambda r)}{\{i\sigma(1-\beta\lambda^2) - \lambda^2\} K_{\frac{3}{2}}(a\lambda)} \left[a + \epsilon N(Z) \left\{\frac{5}{2} + \frac{2a\lambda K_{\frac{1}{2}}(a\lambda) + 3K_{\frac{3}{2}}(a\lambda)}{2K_{\frac{3}{2}}(a\lambda)}\right\}\right] \qquad \cdots \quad (32)$$

From Fourier Integral Theorem,

If $\quad \phi(\eta) = \int_{-\infty}^{\infty} e^{-i\eta T} \bar{\phi}(T) dT$

Then $\quad \bar{\phi}(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\eta T} \cdot \phi(\eta) d\eta \qquad \cdots \quad (33)$

Using this notation, we have

$$W(r, T)=\frac{\Omega a^{\frac{3}{2}}}{2\pi}\int_{-\infty}^{\infty}\frac{2\lambda e^{\lambda^2 T}K_{\frac{3}{2}}(\lambda r)}{\{\sigma(1-\beta\lambda^2)+i\lambda^2\}K_{\frac{3}{2}}(a\lambda)}$$

$$\left[a+\epsilon N(z)\left\{\frac{5}{2}+\frac{2a\lambda K_{\frac{1}{2}}(a\lambda)+3K_{\frac{3}{2}}(a\lambda)}{2K_{\frac{3}{2}}'(a\lambda)}\right\}\right]\frac{d\lambda}{(\beta\lambda^2-1)} \quad \cdots \quad (34)$$

Again, using the substitution $T=\alpha t$, $W(r, T)=r^{\frac{3}{2}}\omega(r, t)$,

we get,

$$\omega(r, t)=\frac{\Omega a^{\frac{3}{2}}}{2\pi r^{\frac{3}{2}}}\cdot\int_{-\infty}^{\infty}\frac{2\lambda e^{\alpha t\lambda^2}K_{\frac{3}{2}}(\lambda r)}{\{\sigma(1-\beta\lambda^2)+i\lambda^2\}K_{\frac{3}{2}}(a\lambda)}\Bigg[a+\epsilon N(z)$$

$$\left\{\frac{5}{2}+\frac{2a\lambda K_{\frac{1}{2}}(a\lambda)+3K_{\frac{3}{2}}(a\lambda)}{2K_{\frac{3}{2}}(a\lambda)}\right\}\Bigg]\frac{d\lambda}{(\beta\lambda^2-1)} \quad \cdots \quad (35)$$

For the order $n+\frac{1}{2}$, where $n\epsilon I$, the Bessel function and modified Bessel functions both reduce to finite form. Thus with the help of the result (Erderlyi, 1953, p. 10)

$$K_{n+\frac{1}{2}}(\lambda)=\left(\frac{\pi}{2\lambda}\right)^{\frac{1}{2}}e^{-\lambda}\sum_{r=0}^{n}(2\lambda)^{-r}\ \frac{\sqrt{n+r+1}}{\lfloor r\ \lfloor n+1-r} \quad \cdots \quad (36)$$

we get,

$$\omega(r, t)=\frac{\Omega a^{\frac{3}{2}}}{2\pi r^{\frac{3}{2}}}\cdot\int_{-\infty}^{\infty}\frac{(\lambda r+1)2\lambda e^{\alpha t\lambda^2-\lambda(r-a)}}{\{\sigma(1-\beta\lambda^2)+i\lambda^2\}(a\lambda+1)}$$

$$\left[a+\epsilon N(z)\left\{\frac{5}{2}+\frac{2a^2\lambda+3(a\lambda+1)}{2(a\lambda+1)}\right\}\right]\frac{d\lambda}{(\beta\lambda^2-1)} \quad \cdots \quad (37)$$

The result for rough sphere is also valid for the slow rotations of the sphere, in that case external force does not exist and the pressure remains constant throughout the liquid.

The Author is thankful to Dr. Har Swarup Sharma, Department of Mathematics, Agra College, Agra for his kind help.

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Spontaneous symmetry breaking in a supersymmetric model

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(*Received for publication in July, 1977*)

[**Abstracts :** It has been demonstrated that in the supersymmetric model of Wess and Zumino the vacuum expectation values of all fields vanish. Consequently, the model is unable to support vacuum instability. Also, no unreasonable mass-spectrum in the form of tachyons in the model is present.]

Introduction

The foundation of supersymmetric models was laid by Schwinger[1] in his investigation on transformations whose generators transform like spinors of odd rank with respect to Lorentz transformation. Recently, a renormalizable field theoretical model concerning spinorial groups have been advanced by Wess and Zumino[2, 3, 4]. The Wess-Zumino (WZ) supersymmetric model is renormalizable in the sense that to all orders in perturbation theory the Ward identities following from supersymmetry transformation are satisfied[3]. Two of the original motivations of the WZ supersymmetric model were to see whether the model leads to a superrenormalizable theory or a non-renormalizable theory becomes renormalizable when supersymmetry is imposed, and whether such models account a theoretical justification for the existence of neutrino in nature[5]. For the second motivation the idea is to look for spontaneous symmetry breaking in the theory corresponding to the transformation

$$\Psi \rightarrow \Psi + \epsilon F. \qquad \cdots \qquad (1)$$

Here F is suitable operator (a Lagrange multiplier field in the WZ model), scalar with respect to Lorentz transformation, and

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Intrcduction

The foundation of supersymmetric models was laid by Schwinger[1] in his investigation on transformations whose generators transform like spinors of odd rank with respect to Lorentz transformation. Recently, a renormalizable field theoretical model concerning spinorial groups have been advanced by Wess and Zumino[2, 3, 4]. The Wess-Zumino (WZ) supersymmetric model is renormalizable in the sense that to all orders in perturbation theory the Ward identities following from supersymmetry transformation are satisfied[3]. Two of the original motivations of the WZ supersymmetric model were to see whether the model leads to a superrenormalizable theory or a non-renormalizable theory becomes renormalizable when supersymmetry is imposed, and whether such models account a theoretical justification for the existence of neutrino in nature[5]. For the second motivation the idea is to look for spontaneous symmetry breaking in the theory corresponding to the transformation

$$\Psi \to \Psi + \epsilon F. \qquad \cdots \qquad (1)$$

Here F is suitable operator (a Lagrange multiplier field in the WZ model), scalar with respect to Lorentz transformation, and

ϵ is an internal spinorial variable, a Majorana spinor whose components are constant anticommutating numbers. However, it appears that it is still an open question whether WZ supersymmetric model does permit spontaneous symmetry breaking[3, 6, 7].

In this paper an attempt has been made to impart a close look on the possibility of spontaneous breakdown in the WZ supersymmetric model. Three primary notions for the existence of a Goldstone particle in the theory have been executed. These are : (*a*) the theory must be manifestly covariant, (*b*) the non-vanishing vacuum expectation value of a scalar field should indicate the presence of a Goldstone particle, and (*c*) the Goldstone field must be coupled to the trace of the energy momentum tensor. In view of the above three constraints, it appears that in the tree approximation the $m\neq 0$, interacting, WZ supersymmetric model does not support vacuum instability. Also, there is no possibility for the emergence of tachyons in the model. This has been dictated in the model by the vacuum stability condition which is a built-in feature in the model.

Preliminaries

The Lagrangian for the massive, interacting WZ supersymmetric model is of the following form :

$$\begin{aligned} L = & -\frac{1}{2}\Big[\partial_\mu A\partial^\mu A+\partial_\mu B\partial^\mu B+i\bar{\psi}\gamma_\mu\partial^\mu\psi-F^2-G^2\Big] \\ & +m\Big[FA+GB-\frac{i}{2}\bar{\psi}\psi\Big] \\ & +g[FA^2-FB^2+2GAB-i\,\bar{\psi}\,\psi B-i\,\bar{\psi}\,\gamma_5\psi B] \qquad (2) \end{aligned}$$

Here A s are scalar fields, B s are pseudoscalar fields, ψ s are Majorana spinors, and F (scalar) and G (pseudoscalar) are two Lagrange multiplier fields. Under an infinetesimal supersymmetry transformation these fieids transform in the following ways :

$$\begin{aligned} & \delta A=[\bar{\epsilon}Q, A]=i\,\bar{\epsilon}\,\psi, \\ & \delta B=[\bar{\epsilon}Q, B]=i\,\bar{\epsilon}\,\gamma_5\,\psi, \\ & \delta\psi=[\bar{\epsilon}Q, \psi]=\partial_\mu(A-\gamma_5 B)\gamma^\mu\epsilon+(F+\gamma_5 G)\epsilon, \\ & \delta F=[\bar{\epsilon}Q, F]=i\,\bar{\epsilon}\,\gamma_\mu\partial^\mu\psi, \\ & \delta G=[\bar{\epsilon}Q, G]=i\,\epsilon\,\gamma_5\gamma^\mu\partial_\mu\psi_o. \qquad \cdots \qquad (3) \end{aligned}$$

The anticommutating Majorana spinors obey

$$[\bar{\epsilon}_1\psi, \bar{\psi}\epsilon_2]=\bar{\epsilon}_1\gamma_\mu\epsilon_2 P^\mu,$$

where ϵ_1 and ϵ_2 anticommute with each other and with ψ. The equation of motion following from the Lagrangian (2) are

$$\Box A+mF+2gFA+2gGB-ig\bar{\psi}\,\psi=0,$$

$$\Box B+mG-2gFB+2gGA+ig\bar{\psi}\gamma_5\psi=0,$$

$$\gamma^\mu\partial_\mu\psi+m\psi+2g(A-\gamma_5 B)\psi=0,$$

$$F+mA+g(A^2-B^2)=0,$$

and $$G+mB+2gAB=0. \qquad \cdots \qquad (4)$$

The last two of the equations (4) do not involve any dynamical operator and these two equations of motion for the auxiliary fields F and G can be used to eliminate them from the Lagrangian (2) which then takes the form

$$\begin{aligned} L'=&-\frac{1}{2}\partial_\mu A\partial^\mu A-\frac{1}{2}\partial_\mu B\partial^\mu B \\ &-\frac{i}{2}\bar{\psi}\gamma_\mu\partial^\mu\psi-\frac{1}{2}m^2A^2-\frac{1}{2}m^2B^2 \\ &-\frac{i}{2}m\bar{\psi}\psi-gmA(A^2+B^2) \\ &-\frac{1}{2}g^2(A^2+B^2)^2-ig\bar{\psi}(A-\gamma_5 B)\psi \qquad \cdots \qquad (5)\end{aligned}$$

For a renormalizable supersymmetric theory the Lagrangian (2) can atmost have another additive term λF where λ is a constant. This implies that the Lagrangian

$$(2)-\lambda F \qquad \cdots \qquad (6)$$

under transformation (3) changes with a total divergence yielding a conserved Noether current

$$\begin{aligned} J_\mu=&\gamma_\lambda\partial^\lambda(A-\gamma_5 B)\gamma_\mu\psi \\ &+m\gamma_\mu(A-\gamma_5 B)\psi \\ &+g\gamma_\mu(A-\gamma_5 B)^2\psi \\ &+\lambda\gamma_\mu\gamma_5\psi. \qquad \cdots \qquad (7)\end{aligned}$$

The equation of motion following from the Lagrangian (6) are the same (except for F) as Eqs. (4). However, the equation of motion for F following from (6) is

$$F+mA+g(A^2-B^2)-\lambda=0 \qquad \cdots \qquad (8)$$

Spontaneous symmetry breaking

We can associate the spontaneous breakdown of supersymmetry with a conserved second rank tensor current[8] and if the theory is manifestly covariant there will emerge Goldstone neutrinos. Let us consider an irreducible set of (massive) Heisenberg fields $\{\psi^{(\lambda)}(x)$ and let

$$\theta^{\mu\nu}(x) \equiv \theta^{\mu\nu}\left[\psi^{(\lambda)}(x)\right]$$

where λ is a set of Lorentz indices. We must also have

$$\partial_\mu \theta^{\mu\nu} = 0 \qquad \cdots \qquad (9)$$

For the spontaneous breakdown to be operative in the theory we should have for some λ,

$$\left\langle \left[\theta^{\mu\nu}(x), \quad \psi^{(\lambda)} x)\right] \right\rangle_o \neq 0 \qquad \cdots \qquad (10a)$$

and

$$<\psi^{(\lambda)}(x)>_o = 0 \qquad \cdots \qquad (10b)$$

are satisfied simultaneously. In equation (10a) and (10b) the vacuum $|\,0>$ is assumed to be unique. Under these conditions, the Goldstone-Salam-Weinberg[9] argument applies to the current $\theta^{\mu 0}$, $\theta^{\mu 1}$, $\theta^{\mu 2}$, and $\theta^{\mu 3}$ separately and the massless particles produced in these channels transform respectively as the 0, 1, 2 and 3 components of a spinor (Majorana).

A familiar way of stating Eq. (10) is

$$\begin{aligned} 0 &\neq <\delta\psi_\alpha>_o \\ &= <[\textstyle\int d\overset{3}{x}\, \bar{\epsilon}^{\beta}\, \theta_{o\beta}(x), \psi_\alpha(x')]>_o \\ &= <F>_o\, \epsilon_\alpha \\ &= <F>_o\, C_{\alpha\beta}\, \bar{\epsilon}^{\beta}. \end{aligned} \qquad \cdots \qquad (11)$$

This formal expression is the result of the action of an infinetesimal supersymmetry transformation ϵ_α on the spinor (see Eq. (3)) and $C_{\alpha\beta}$ is the charge conjugation matrix. In Eq. (11) $<F>_o$ is the vacuum expectation value of the auxiliary

scalar field connected to ψ by supersymmetry transformation (3). We can write the Ward identity

$$\partial^{\mu} < T^{*}\theta_{\mu\alpha}(x)\psi_{\beta}(o) >_{o} = C_{\alpha\beta} < F >_{o} \delta_{4}(x) \quad \cdots \quad (12)$$

Introducing the momentum space Fourier transformation for the amplitude in (12) we resolve it into the following invariant componenets :

$$\int dx\, e^{ikx} < T^{*}\theta_{\mu\alpha}(x)\psi_{\beta}(o) > = M_{1}(k^{2})K_{\mu}C_{\alpha\beta} + (M_{2}(k^{2})\eta_{\mu\nu} + M_{3}(k^{2})k_{\mu}k_{\nu})(\gamma^{\nu}C)_{\alpha\beta} + M_{4}(k^{2})k^{\nu}(\sigma_{\mu\nu}C)_{\alpha\beta}, \quad \cdots \quad (13)$$

and then we use Eq. (12) to get

$$M_{1} = \frac{< F >^{o}}{k^{2}} \quad \cdots \quad (14)$$

The presence of such a zero-mass pole with residue $< F >_{o}$ indicates the emergence of a Goldstone spinor[6].

Two apparent contradictions

One can show that there are two apparent inconsistencies in having a Goldstone spinor in the WZ supersymmetric model. The first one is that in this model the supersymmetric charge does annihilate the vacuum[8].

Since supersymmetry generators are spinors with respect to the Lorentz group to retain correct relation between spin and statistics we consider anticommutators between such generators, for example,

$$\left\{ Q_{\alpha}, \bar{Q}_{\beta} \right\} = -2\gamma^{\mu}_{\alpha\beta} P_{\mu}, \quad \cdots \quad (15)$$

where $\alpha, \beta = 0, 1, 2, 3$. Along with Eq. (15) the infinetesimal supersymmetry algebra is given by

$$[P_{\mu}, P_{\nu}] = 0, \quad \cdots \quad (16a)$$

$$[P_{\mu}, Q_{\alpha}] = 0, \quad \cdots \quad (16b)$$

and

$$\left[P_{\mu}, [P_{\lambda}, [P_{\nu}, [Q_{\alpha}]] \right]. \quad \cdots \quad (16c)$$

One noteworthy point is that in the WZ supersymmetric model along with the scalar constant motion, *i.e.*, weak and electromagnetic charges, and vector constant motion, *i.e.*, four momentum operator also are present the *spinor constant of motion* Q_α.

Now multiplying Eq. (15) with γ^λ and taking the trace with respect to the spinor indices, we get

$$P^\lambda = -\frac{1}{8}\sum_{\alpha,\,\beta}\left\{ Q_\alpha,\ Q_\beta\right\}\left(\gamma^o\gamma^\lambda\right)_{\alpha,\beta.} \quad \cdots \quad (17)$$

From Eq. (17) for the Hamiltonian $H = P_o$ we can write

$$H = P_o = \frac{1}{4}\sum_\alpha Q_\alpha^2 \quad \cdots \quad (18)$$

But in the WZ supersymmetric model always is true that

$$<Q_\alpha^2>_o = 0 \quad \cdots \quad (19)$$

i.e.,

$$Q\,|\,o> = 0 \quad \cdots \quad (20)$$

In getting (19) one uses the Ward identity[3]

$$<AA>_o = <BB>_o \quad \cdots \quad (21)$$

associated with the supersymmetry transformation. At the same time what is so interesting is that the supersymmetry transformation of the type (1) attributes us a responsibility to inquire the possibility of spontaneous breakdown in the theory. One then arrives at some highly unaesthetic feature. It is that computed particle masses corresponding to two sets of ground states (see the table below) show that there are tachyons in the model.

The Euler Lagrange Eqs. for F and G are given by Eqs. (8) and (4) which are actually constraints in the model. The two constraints can be used to eliminate F and G from the Lagrangian (6). We then get a new Lagrangian L'' with the potential $P(A, B)$ and this is

$$\begin{aligned} L'' = & -\frac{1}{2}\partial_\mu A\partial^\mu A - \frac{1}{2}\partial_\mu B\partial^\mu B \\ & -\frac{i}{2}\bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi \\ & -ig\bar{\psi}\,(A - \gamma_5 B\,\psi \\ & + P(A, B) \end{aligned} \quad \cdots \quad (22)$$

where
$$P(A, B) = -\frac{1}{2}[mA + g(A^2 - B^2) - \lambda)^2$$
$$-\frac{1}{2}[mB + 2ABg]^2. \quad \cdots \quad (23)$$

An important remark is in order. A shift in the field A, for example, by putting $A = A + a$ in the Lagrangian (6) eliminates the additional term $-\lambda F$ from Eq. (6) provided

$$ga^2 + ma - \lambda = 0 \quad \cdots \quad (24)$$

is satisfied. For a real solution of the above quadratic equation one must have

$$m^2 + 4\lambda g \geqslant 0 \quad \cdots \quad (25)$$

We now find[10] the extreema of $P(A, B)$. The ground states thus obtained should be tested for stability by computing particle masses. The values of $<A>_o$, $<B>_o$, $<F>_o$ and $<G>_o$ thus obtained are stated below, also are given squares of masses m_A, m_B, and m_ψ.

We have three solutions S_1, S_2 and S_3, corresponding to $m^2 > -4\lambda g$, $m^2 < -4\lambda g$ and $m^2 = -4\lambda g$ respectively. Of these, S_1 and S_2 are symmetry preserving, $<F>_o = 0$. But corresponding to the Goldstone solution $<F>_o \neq 0$, computed particle masses show that either A or B is a tachyon. Also, corresponding to the symmetry preserving solution S_3 computed particle masses show that A, B and ψ are all tachyons (See Table).

Resolution of the paradox

Let us now check whether the Goldstone field F is at all coupled to the trace of the energy momentum tensor. The canonical energy momentum tensor in this model is

$$T_{\mu\nu} = \partial_\mu A \partial_\nu A - \partial_\mu B \partial_\nu B$$
$$+ \frac{i}{4}(\overline{\psi}\gamma_\mu \gamma_\nu \psi + \bar{\psi}\gamma_\nu \partial_\mu \psi)$$
$$+ \eta_{\mu\nu} L. \quad \cdots \quad (26)$$

To give it a nicer trace, following standered recipe, an improved energy momentum tensor may look like this[3] :

$$\theta'_{\mu\nu} = T_{\mu\nu} - \frac{1}{6}(\partial_\mu \partial_\nu - \eta_{\mu\nu}\Box)(A^2 + B^2), \quad \cdots \quad (27)$$

and then

$$\theta'^{\mu}_{\mu}=m\left(FA+GB-\frac{i}{2}\bar{\psi}\psi\right). \qquad \cdots \qquad (28)$$

A further modification leads to

$$\theta_{\mu\nu}=\theta'_{\mu\nu}-\frac{m}{6g}(\partial_\mu\partial_\nu-\eta_{\mu\nu}\Box)A \qquad \cdots \qquad (29)$$

so that

$$\theta^{\mu}_{\mu}=-\frac{m^2}{2g}F. \qquad \cdots \qquad (30)$$

This shows that the Lagrangian multiplier field F is coupled to the trace of the energy momentum tensor and

$$\left\langle \theta^{\mu}_{\mu} \right\rangle_0=-\frac{m^2}{2g}\langle F\rangle_0.$$

The choice (29) in the model is a good one, since its trace (30) has the same form when expressed in terms of renormalized quantities. It has been observed that the modified form of the energy momentum tensor $\theta'_{\mu\nu}$ can be obtained from the form $\theta'_{\mu\nu}$ [Eq. (27)] by the replacement

$$A\to A+\frac{m}{2g}\cdot \qquad \cdots \qquad (31)$$

Under this replacement Lagrangian (2) has a form where the bilinear mass term is absent in the model and has actually been replaced by the term $-\left(\frac{m^2}{4g}\right)F$.

We shall now argue that the relation is so sacred that it actually forbids any of the fields to procure a non-zero vacuum expectation value. Let us decompose F into its positive and negative frequency parts $F=F^{+}-F^{-}$. Since by Eq. (20) the vacuum is an eigenstate of the charge operator. *i.e.*, $Q\mid 0\rangle 0$

also since[11]

$$QF^{-}\mid 0\rangle=gF^{-}\mid 0\rangle$$

we have

$$F^{-}\mid 0\rangle=g^{-1}[Q,F^{-}]\mid 0\rangle \qquad \cdots \qquad (32)$$

Table

	$<A>_0$	$<B>_0$	$<F>_0$	$<G>_0$	m_A^2	m_B^2	m_ψ^2
S_1	$-\frac{m}{2g}$	$\pm\frac{im}{2g}\sqrt{1+\frac{4\lambda g}{m^2}}$	0	0	$m^2+4\lambda g$	$m^2+4\lambda g$	$2(m^2+4\lambda g)$
S_2	$-\frac{m}{2g}\left\{1\pm\sqrt{1+\frac{4\lambda g}{m^2}}\right\}$	0	0	0	$-(m^2+4\lambda g)$	$-(m^2+4\lambda g)$	$-2(m^2+4\lambda g)$
S_3	$-\frac{m}{2g}$	0	$\lambda+\frac{m^2}{4g}$	0	$\frac{1}{2}(m^2+4\lambda g)$	$-\frac{1}{2}(m^2+4\lambda g)$	0

In view of transformation (3) we can at once write

$$<\delta F^{-}>_0 = g^{-1} <[\bar{\epsilon} Q, F^{-}]>_0$$
$$= ig^{-1} \bar{\epsilon} \gamma_\mu < \partial^\mu \psi^{-} >_0$$
$$= 0 \qquad \ldots \qquad (33)$$

Similar procedure can be taken for the F^{+} part. This amounts simply to the fact that

$$<F>_0 = 0 \qquad \ldots \qquad (34)$$

so that we can put

$$\frac{m^2}{4g} + \lambda = 0 \qquad \ldots \qquad (35)$$

i.e.,

$$\lambda = -\frac{m^2}{4g} \qquad \ldots \qquad (36)$$

From the table already given for the vacuum expectation values for different fields it appears that at this value of λ,

$$<F>_0 = <B>_0 = <G>_0 = 0$$

and $$m_A^2 = m_B^2 = m_\psi^2 = 0$$

but $$<A>_0 = \frac{m}{2g} \neq 0 \qquad \ldots \qquad (37)$$

But once again the field can be decomposed into A^{+} and A^{-} and the simple technique adopted for F can also be used here to show that

$$<\delta A^{\pm}>_0 = g^{-1} <[\bar{\epsilon} Q, A^{\pm}]>_0 \qquad \ldots \qquad (38)$$

$$= ig^{-1} \bar{\epsilon} <\psi^{\pm}>_0 = 0,$$

Thus in the WZ supersymmetric model the vacuum expectation values of all fields vanish.

Further comments

What we have been dealing so far was that the WZ supersymmetric model in the tree approximation could not support a Goldstone potential. Two important perspectives are in order. First, is the above conclusion true in any order above the zeroth one? Second, to what extent one is supported theoreti-

cally if desired to avoid the emergence of Goldstone neutrinos ? Before coming to these questions theoretically let us take the problem of spontaneous symmetry breaking to some extent formally,

Let us define an automorphism α of the field algebra f, commuting with proper Poincare′ group, by a set of transformation $(i=1,\cdots,N)$

$$A_i \rightarrow A_i^{\alpha} \qquad \cdots \qquad (39)$$

and suppose that this transformation corresponds to an n-parametric continuous group G of transformation of fields

$$A_i(x) \rightarrow A_i^{g(\lambda)}(x) \qquad \cdots \qquad (40)$$

where $g(\lambda)=g(\lambda_1,\ldots,\lambda_n)\epsilon G$.

If the generator of (40) is some n local conserved vector currents J_μ, then its relation with $A_i(x)$ may be represented in this way :

$$\frac{d}{d\lambda} A_i^{g(\lambda)}(x')=i\left[\int d^3x f_R(\underset{\smile}{x}) j_a^o(\underset{\smile}{x}, t),\ \phi_i(x)\right] \qquad \cdots \qquad (41)$$

where $f_R(x)$ is some test function which is 1 for $|\underset{\smile}{x}|<R$ and vanishes for $|\underset{\smile}{x}|>R+\epsilon$. Now if the theory involves spontaneous symmetry breaking, the Goldstone theorem is stated thus :

$$\lim_{R\rightarrow\infty} <\Omega \mid \left[\ Q^a_{R,t},\ A\right] \mid \Omega>$$

$$=\lim_{R\rightarrow\infty} <\Omega \mid Q^a_{R,t}\ E_1 A - A E_1 Q^a_{R,t} \mid \Omega> \neq 0 \qquad \cdots \qquad (42)$$

where $Q^a_R(t)=\int d^3x f_R(\underset{\smile}{x}) j^o_a(\underset{\smile}{x}, t)$

A is any element of the field algebra f, E_1 is the projection on the zero-mass one particle state and Ω is the true vacuum.

If the R.H.S. of (42) is equal to zero E_1 does not exist and the Goldstone theorem fails.

In such a case we always have

$$A_i^a = V_a\, A_i\, V_a^{-1} \qquad \cdots \qquad (43)$$

where α is an exact symmetry. If the transformation involved are continuous then

$$V_\alpha = \exp[i\alpha Q]$$

and the generator Q is also conserved. In the WZ supersymmetric model the unitary operator V_α does exist and α is not spontaneously broken. Thus it follows that the automorphism α being exact possesses some required *continuity properties* so that it can be implemented globally[12]. Spontaneous breakdown of local α does not exist unless electrons are free of electromagnetic interactions and are massless[13]. Let us consider the case of local automorphism. The zeroth order potential for a renormalisable supersymmetric model from a scalar multiplet (consisting of a Majorana spinor ψ, a complex scalar field F, and a complex scalar field A) can be written as

$$V(A) = F_a^+ (A) F_a(A)$$

with $$F_a(A) = \lambda_a + m_{ab} A_b + g_{abc} A_b A_c,$$

where λ, m and g are parameters occuring in the Lagrangian. If there is no zeroth order spontaneous breakdown of supersymmetry there exists points $A_a = A$ which satisfy $(V(A) \geqslant 0)$

$$F_a^+(A) = 0 \qquad \cdots \qquad (44)$$

This amounts to that the R. H. S. of (42) should then always be zero.

Formal arguments on the question of spontaneous breakdown of supersymmetry leads to the conclusion that first order radiative correction does not invoke spontaneous breakdown in the WZ supersymmetric model[14]. However, the problem actually depends on how strongly the scalar field self-couples.

Let us consider an $m=0$ Lagrangian in this form :

$$L = \frac{1}{2}(\partial A)^2 + \frac{1}{2}(\partial B)^2 + \frac{1}{2}\bar{\psi} i \psi - \frac{g}{2}\bar{\psi}(A - i\gamma_5 B)\psi$$
$$- \frac{\lambda}{8}(A^2 + B^2).$$

The model is supersymmetric when $\lambda = g^2$ and the potential[15] is (ϕ is some expectation value A) :

$$V(\phi) = \frac{\phi^4}{8}\left\{\lambda + \frac{\hbar}{16\pi^2}\left(5\lambda^2 - 4g^4\right)\left(\ln\frac{\phi^2}{M^2} - \frac{25}{6}\right) + o(\hbar^2)\right\}$$

M is a suitable scale parameter introduced to define λ, $V(\phi)$ is reliable[15] for $\lambda \leqslant 1$, $g \leqslant 1$, $\lambda^2 \ln(\phi^2/M^2) \leqslant 1$, and $g^4 \ln(\phi^2/M^2) \leqslant 1$ and if $\lambda \approx g^4 \leqslant 1$, $V(\phi)$ does not have a minimum. Thus when first order radiative correction is introduced the problem of vacuum stability will depend on the relative magnitude of scalar self-coupling λ and fermion scalar coupling g. When $\lambda = g^2$ there is no vacuum instability.

If neutrino is a Goldstone particle then supersymmetry involves some serious unaesthetic affair[16]. Its emergence as a Goldstone particle is incompatible to experimental facts on the spectrum in beta decay[5].

Zumino has pointed out three possible ways to circumvent such difficulties. These are :

(*a*) some anomally (like that in $\pi^\circ \Rightarrow 2\gamma$) invalidates the trouble with the beta spectrum,

(*b*) there is no spontaneous symmetry breaking but there is explicit soft breakdown giving relations between renormalized quantities,

(*c*) supersymmetric theory of gravitation should be a way out.

However, ν_e and ν_μ can be obtained (avoiding Goldstone mechanism) from a set of phase and γ_5 transformations connected with lepton number conservation[17].

Conclusion

The observation that the WZ supersymmetric model does not support vacuum asymmetry should be regarded as a novel feature of the model itself. This has actually been dictated by the fact $Q \mid 0 \rangle = 0$, and thus $P_\mu \mid 0 \rangle = 0$, so that there is no violation of Lorentz invariance and no emergence of unreasonable mass-spectrum in the theory. It is noteworthy that the very feature of the WZ supersymmetric model, $Q \mid 0 \rangle = 0$ is sufficient

to guarantee that in this model the sum of all vacuum diagrams vanishes identically as a consequence of the compensation among contributions involving different fields of the supersymmetry multiplet[18],

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